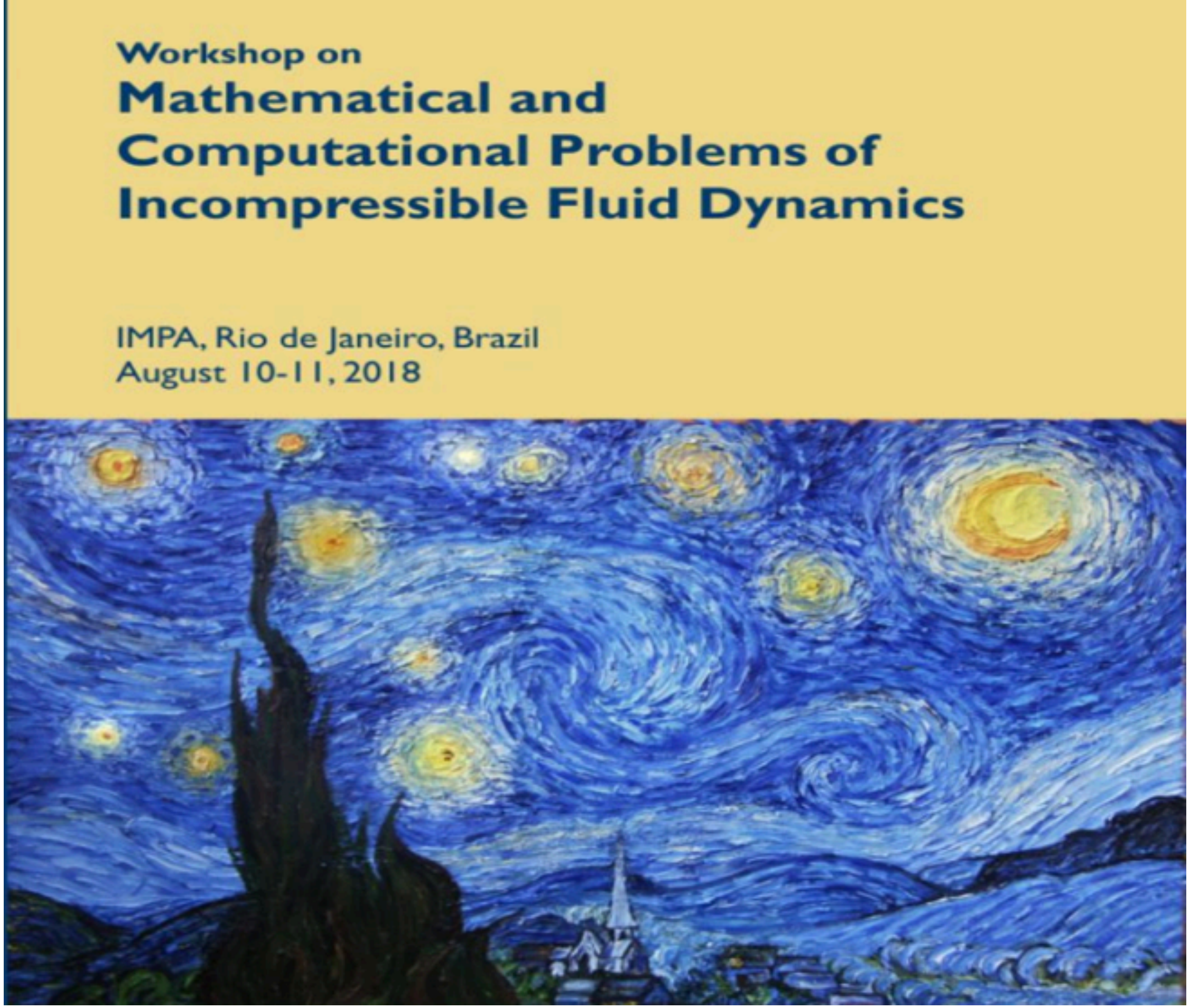


Walking droplets correlated at a distance

André Nachbin, IMPA, Brazil

problem due to a WAVE-PARTICLE ASSOCIATION

YCouder & EFort

= We look at an object never studied before

= Never imagined to exist in Classical Mechanics

= We formulated the first PDE system for a hydrodynamic pilot-wave

J. Fluid Mech. (2015), vol. 778, pp. 361–388. © Cambridge University Press 2015 361
doi:10.1017/jfm.2015.386

Faraday pilot-wave dynamics: modelling and computation

Paul A. Milewski[1,†], Carlos A. Galeano-Rios[2], André Nachbin[2] and John W. M. Bush[3]

= We compare with laboratory experiments: J.Bush@MIT

>>>> Problem with great potential & need for Math contributions

Physical setup

John Bush (MIT/Math; Wet Lab), @Discovery Channel;

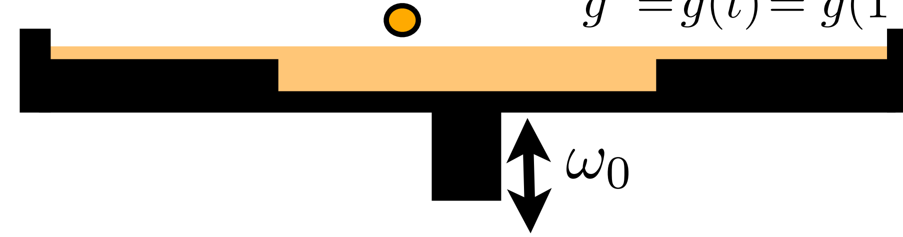
You Tube >> waterdrop shot in 10000 frames a second

"coalescence cascade"

http://www.openculture.com/2010/12/water_drop_filmed_in_10000_frames_per_second.html video by John Bush: a bouncing (silicon oil) droplet

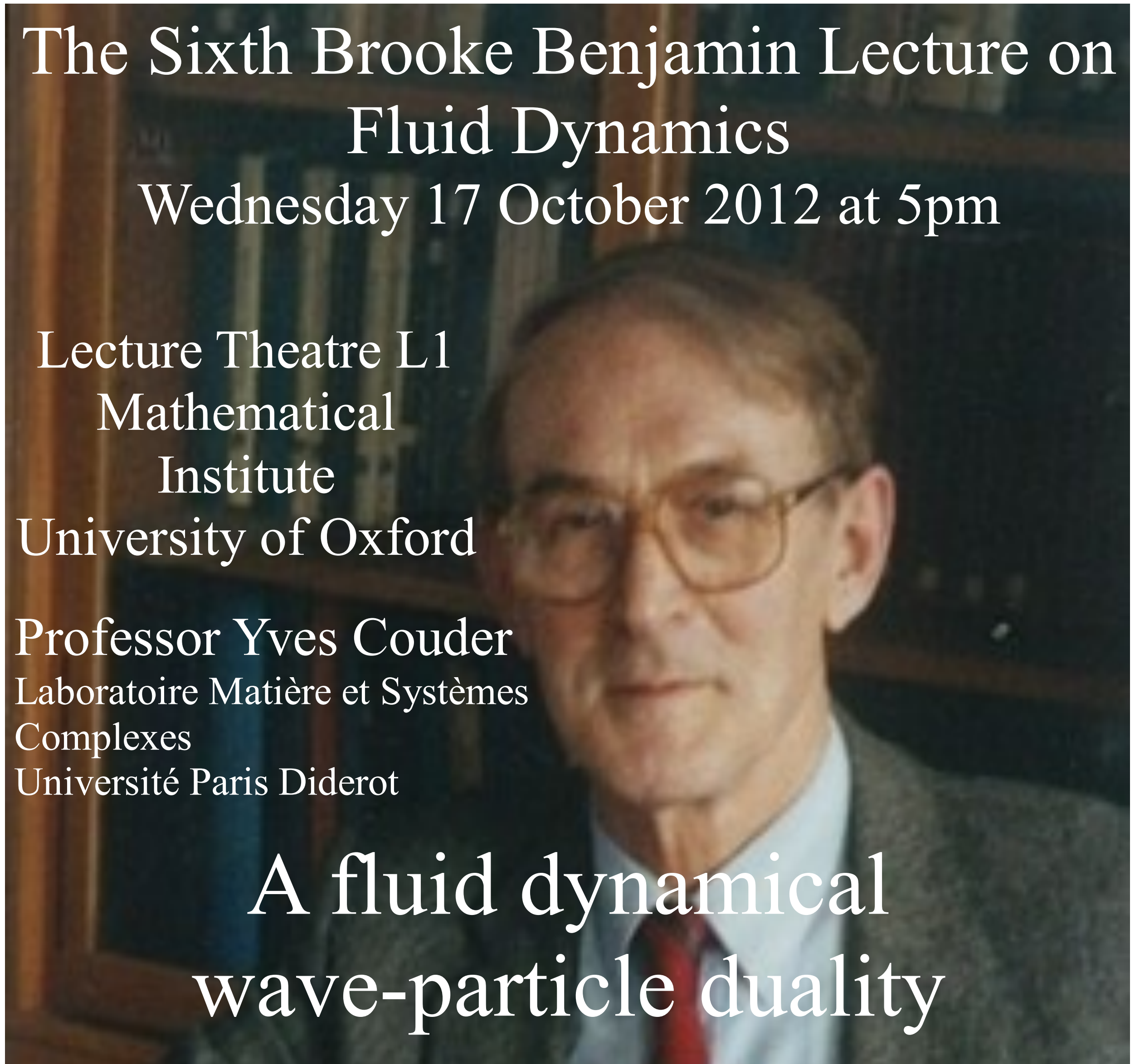
J. Bush/MIT: https://www.youtube.com/watch?v=nmC0ygr08tE

$$g^* = g(t) = g(1 - \Gamma \sin(\omega_0 t))$$



Yves Couder: https://www.youtube.com/watch?v=W9yWv5dqSKk

mechanism: Faraday wave instability >>>>> the PILOT-WAVE



The Sixth Brooke Benjamin Lecture on Fluid Dynamics

Wednesday 17 October 2012 at 5pm

Lecture Theatre L1
Mathematical
Institute
University of Oxford

Professor Yves Couder
Laboratoire Matière et Systèmes
Complexes
Université Paris Diderot

A fluid dynamical wave-particle duality

Wave-particle duality is a quantum behaviour usually assumed to have no possible counterpart in classical physics. We revisited this question when we found that a droplet bouncing on a vibrated bath could become self-propelled by its coupling to the surface waves it excites. A dynamical wave-particle association is thus formed. Through several experiments we addressed the same general question. How can a localized and discrete droplet have a common dynamics with a continuous and spatially extended wave? Surprisingly several quantum-like behaviours emerge; a form of uncertainty and a form of quantization are observed. I will show that both properties are related to the "path memory" contained in the wave field. The relation of this experiment with the pilot-wave models proposed by de Broglie and by Bohm for quantum mechanics will be discussed.

All are warmly invited to attend the lecture and reception that follows.

Please email hicks@maths.ox.ac.uk to register your attendance.

http://www.maths.ox.ac.uk/events/brooke-benjamin-lecture

Yves Couder's lab, Paris 7

A fluid dynamical wave-particle duality

Wave-particle duality is a quantum behaviour usually assumed to have no possible counterpart in classical physics. We revisited this question when we found that a droplet bouncing on a vibrated bath could become self-propelled by its coupling to the surface waves it excites. A dynamical wave-particle association is thus formed. Through several experiments we addressed the same general question. How can a localized and discrete droplet have a common dynamics with a continuous and spatially extended wave? Surprisingly several quantum-like behaviours emerge; a form of uncertainty and a form of quantization are observed. I will show that both properties are related to the "path memory" contained in the wave field. The relation of this experiment with the pilot-wave models proposed by de Broglie and by Bohm for quantum mechanics will be discussed.

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http://www.maths.ox.ac.uk/events/brooke-benjamin-lecture

Y. Couder, S. Protière, E. Fort, and A. Boudaoud, Nature (London) **437**, 208 (2005).

PRL **97**, 154101 (2006) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2006

Single-Particle Diffraction and Interference at a Macroscopic Scale

Yves Couder[1,*] and Emmanuel Fort[2,†]

Pilot-Wave Hydrodynamics

John W.M. Bush

Department of Mathematics, Massachusetts Institute of Technology, Cambridge,

Annu. Rev. Fluid Mech. 2015. 47:269–92

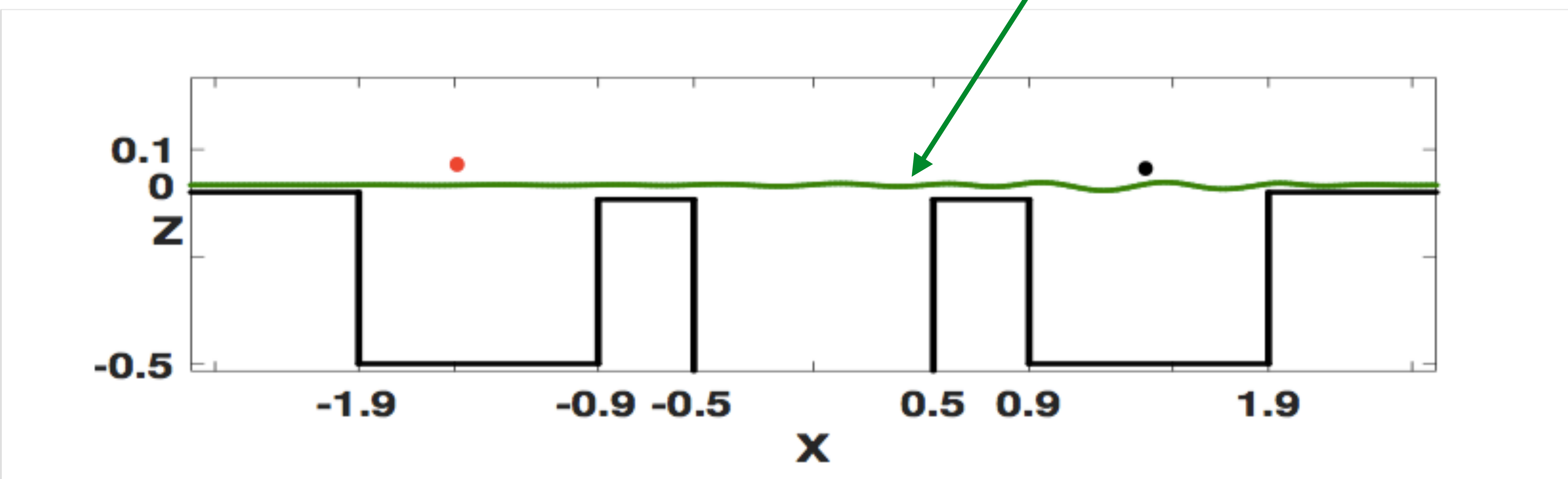
AIP | Chaos
An Interdisciplinary Journal of Nonlinear Science

2018 special issue on HQA, to appear.

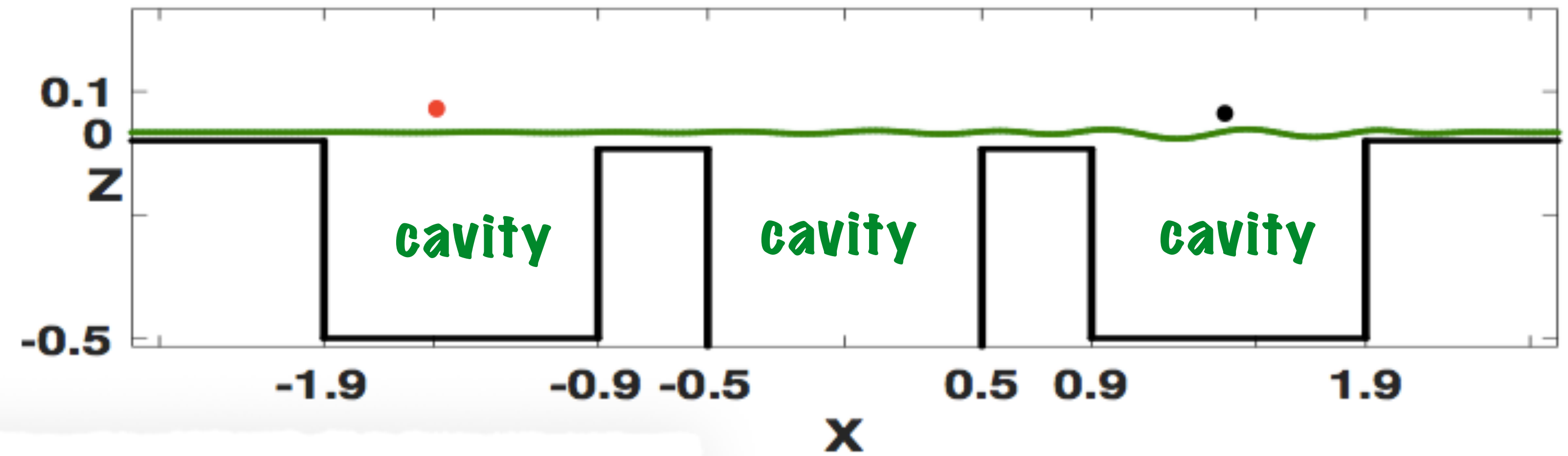
incompressible fluid modelling:

BENJAMIN, T.B. & URSELL, F. 1954
DIAS, F., DYACHENKO, A. I. & ZAKHAROV, V. E. 2008
AMBROSE, D.M., BONA, J.L. & NICHOLLS, D.P. 2012

- * Navier-Stokes eq. with a (linear) free surface: material surf + stress balance
- * Helmholtz decomposition: curl free + weakly div free, velocity field
- * asympt. get >>> weakly viscous, perturbed potential theory
- * plus Dirichlet-to-Neumann op. >>> reduce 1-dimension



1D dynamics of 2 droplets placed at a distance



$$m\ddot{X}_1 + c\,F(t)\dot{X}_1 = -F(t)\,\frac{\partial \eta}{\partial x}(X_1(t),t).$$

2 droplet-dynamics

$$m\ddot{X}_2 + c\,F(t)\dot{X}_2 = -F(t)\,\frac{\partial \eta}{\partial x}(X_2(t),t).$$

BENJAMIN, T.B. & URSELL, F. 1954
DIAS, F., DYACHENKO, A. I. & ZAKHAROV, V. E. 2008
AMBROSE, D.M., BONA, J.L. & NICHOLLS, D.P. 2012

$$\frac{\partial \eta}{\partial t} = \underbrace{\frac{\partial \phi}{\partial z}}_{\text{Fourier int. op.}} + 2\nu\frac{\partial^2 \eta}{\partial x^2},$$

$$\frac{\partial \phi}{\partial t} = -\underbrace{g(t)}_{\text{shaking}}\eta + \frac{\sigma}{\rho}\frac{\partial^2 \eta}{\partial x^2} + 2\nu\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\rho}P_d(x-X_1(t)) - \frac{1}{\rho}P_d(x-X_2(t)),$$

2 wave makers

J. Fluid Mech. (2015), vol. 778, pp. 361–388. © Cambridge University Press 2015 361
doi:10.1017/jfm.2015.386

Faraday pilot-wave dynamics: modelling and computation

simpler than...

Paul A. Milewski[1,†], Carlos A. Galeano-Rios[2], André Nachbin[2] and John W. M. Bush[3]

numerics as...

PHYSICAL REVIEW FLUIDS 2, 034801 (2017)

Tunneling with a hydrodynamic pilot-wave model

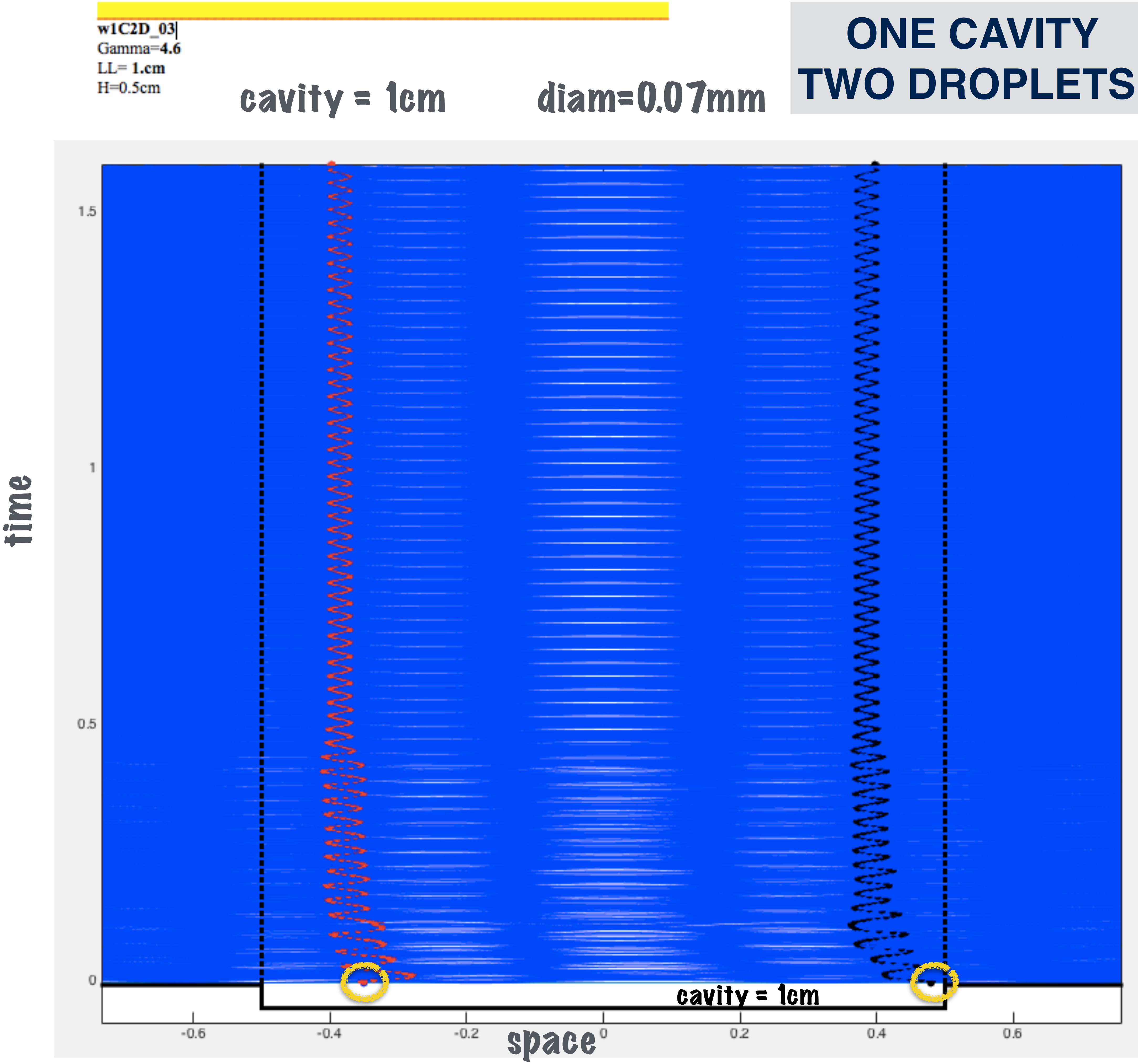
André Nachbin,[1,3] Paul A. Milewski,[2] and John W. M. Bush[3]

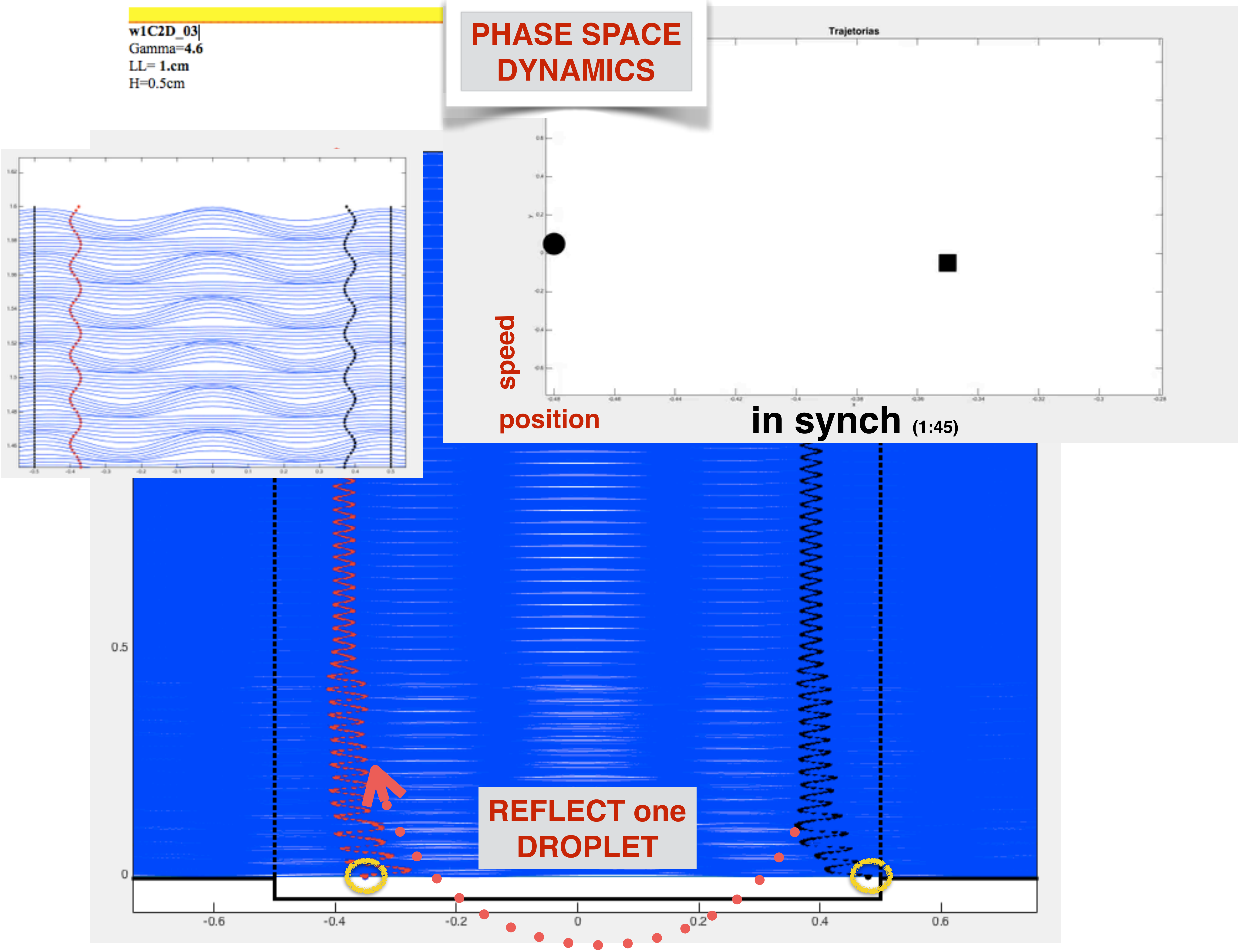
contact time $T_c \equiv T_F/4$ $\qquad g^* = g(t) = g(1-\Gamma\sin(\omega_0 t))$

ONE CAVITY

TWO DROPLETS

ONE CAVITY TWO DROPLETS




w1C2D_03
Gamma=4.6
LL= 1.cm
H=0.5cm
PHASE SPACE DYNAMICS
Trajetorias
speed
position
in synch (1:45)
REFLECT one DROPLET
0.5
0
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

Coupled oscillators that can spontaneously synchonize

limit-cycle coupled oscillators

Kuramoto model:

Winfree'67; Kuramoto '75 (phase transition from incoherence to a coherent state)

$$\dot{\theta}_i = \omega_i + \sum_{j=1}^{N} K_{ij} \sin(\theta_j - \theta_i), \quad i = 1, ..., N,$$

phase nat. freq. coupling matrix nonlinear coupling

O'Keeffe K.P., Hong H., and Strogatz S.H., Nature Comm. **8**, 1504 (2017).

sync and swarm

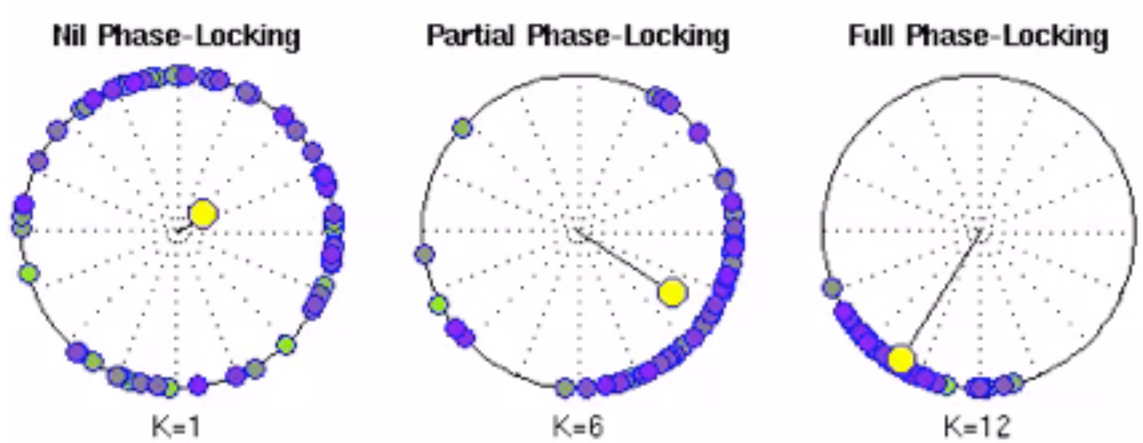
Acebrón J.A., Bonilla L.L., Perez V.C.J., Ritort F., and Spigler R., Rev. Mod. Phys. **77**, 137–185 (2005).

Dorfler F. and Bullo F., Automatica **50**, 1539–1564 (2014).

two nice reviews

Kuramoto Oscillators

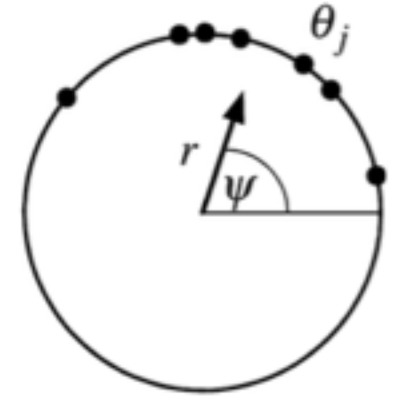
https://en.wikipedia.org/wiki/Kuramoto_model



Nil, partial and full phase-locking in an all-to-all network of Kuramoto oscillators. Phase-locking is governed by the coupling strength K and the distribution of intrinisic frequencies ω. Here, the intrinsic frequencies were drawn from a normal distribution (M=0.5Hz, SD=0.5Hz). The yellow disk marks the phase centroid. Its radius is a measure of coherence.

Kuramoto eq. with identical coupling: $K_{ij} = K$

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i), \quad i = 1, \ldots, N$$



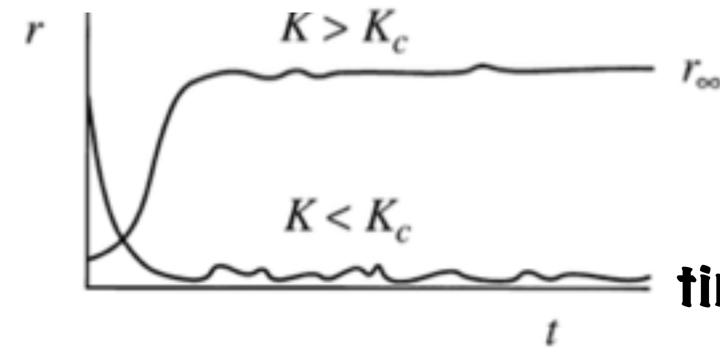
Strogatz, PhysD 2000

$$r\,e^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

order parameter

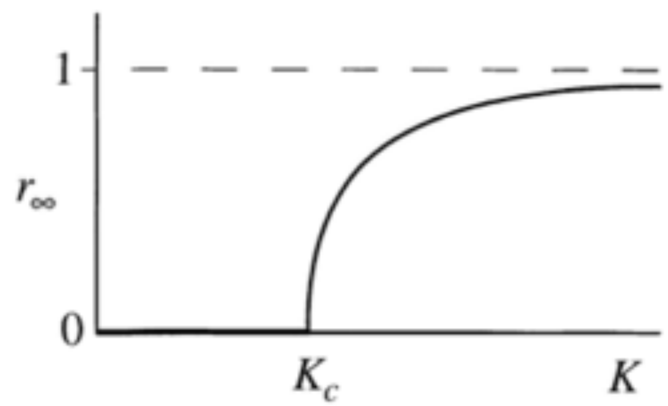
degree of COHERENCE

r=0 : INCOHERENT



time

But when K exceeds K_c, this *incoherent state* becomes unstable and $r(t)$ grows exponentially, reflecting the nucleation of a small cluster of oscillators that are mutually synchronized, thereby generating a collective oscillation. Eventually $r(t)$ saturates at some level $r_\infty < 1$, though still with $O(N^{-1/2})$ fluctuations.



the coherence $r_\infty(K)$ on the bifurcating branch

nonlinear coupling strength

Fig. 3. Dependence of the steady-state coherence r_∞ on the coupling strength K.

This mixed state is often called *partially synchronized*. With further increases in K, more and more oscillators are recruited into the synchronized cluster, and r_∞ grows as shown in Fig. 3.

Found Phys (2011) 41: 843–862
DOI 10.1007/s10701-010-9527-y

Stochastic electrodynamics (**SED**) is an extension of the de Broglie–Bohm interpretation of quantum mechanics, with the electromagnetic zero-point field (ZPF) playing a central role as the guiding pilot-wave. The theory is a deterministic nonlocal hidden-variable theory.[2][3] It is distinct from other more mainstream interpretations of quantum mechanics such as the Copenhagen interpretation and Everett's

wiki

Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field

A. Valdés-Hernández · L. de la Peña · A.M. Cetto

This is just the Abraham-Lorentz equation for the particle located at $\mathbf{x}_i$, with the extra term $\frac{e_i}{e_j} m_j \tau_j \dddot{\mathbf{x}}_j$ representing the force that the radiation reaction of the particle at $\mathbf{x}_j$ exerts on the charge e_i [6].

ZPF

$$m_i \ddot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + m_i \tau_i \dddot{\mathbf{x}}_i + \frac{e_i}{e_j} m_j \tau_j \dddot{\mathbf{x}}_j + e_i \mathbf{E}(\mathbf{x}_i, t).$$

1D theory

$$m_i \ddot{x}_i = f_i(x_i) + m_i \tau_i \dddot{x}_i + e_i \left[E_i(t) + \frac{1}{e_j} m_j \tau_j \dddot{x}_j \right], \tag{12}$$

TWO nearby particles i,j

The electric field is expanded as usual in terms of plane waves

$$\mathbf{E}(\mathbf{x}, t) = \sum_k \tilde{E}(\omega_k) \mathbf{a}_k(x) e^{i\omega_k t} + \text{c.c.},$$

where $\mathbf{a}_k(x)$ is a stochastic vector (with random components)

Consider frequencies, which are determined by the resonant theory (see (7)), as *relevant frequencies*.

Found Phys (2011) 41: 843–862
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Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field

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$$m_i \ddot{\mathbf{x}}_i = \mathbf{f}_i(\mathbf{x}_i) + m_i \tau_i \dddot{\mathbf{x}}_i + \frac{e_i}{e_j} m_j \tau_j \dddot{\mathbf{x}}_j + e_i \mathbf{E}(\mathbf{x}_i, t).$$

1D theory

$$m_i \ddot{x}_i = f_i(x_i) + m_i \tau_i \dddot{x}_i + e_i \left[E_i(t) + \frac{1}{e_j} m_j \tau_j \dddot{x}_j \right], \tag{12}$$

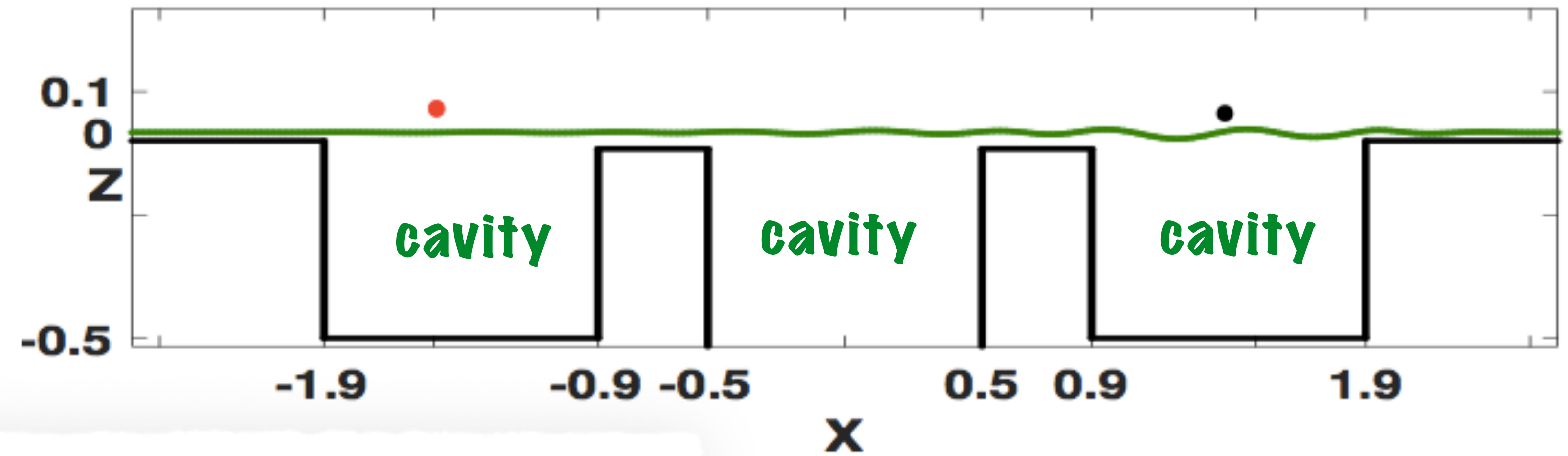
Show that the well defined energy state of the bipartite system, is *non-factorizable* and gives rise to *entanglement*.

It is clear that the possibility of superposing two state vectors to construct a third one is crucial for entanglement to exist. In the usual formalism of quantum mechanics this superposition is understood as a result of the linearity of the Schrödinger equation, whereas in the present context the superposition finds its deeper origin in the relations between the stochastic variables of the common background field,

quantum state of each particle cannot be described independently of the state of the other(s), even when the particles are separated by a large distance—instead, a quantum state must be described for the system as a whole.

wiki

OSCILLATING DROPLETs that can SPONTANEOUSLY SYNC



$$m\ddot{X}_1 + c\,F(t)\dot{X}_1 = -F(t)\,\frac{\partial \eta}{\partial x}(X_1(t),t).$$

$$m\ddot{X}_2 + c\,F(t)\dot{X}_2 = -F(t)\,\frac{\partial \eta}{\partial x}(X_2(t),t).$$

2 droplet-dynamics

IMPLICIT COUPLING: it is a

WAVE-MEDIATED COUPLING THROUGH

a PDE w/ FEEDBACK

contact time $T_c \equiv T_F/4$

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z} + 2\nu\frac{\partial^2 \eta}{\partial x^2},$$

$$\frac{\partial \phi}{\partial t} = -g(t)\eta + \frac{\sigma}{\rho}\frac{\partial^2 \eta}{\partial x^2} + 2\nu\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\rho}P_d(x - X_1(t)) - \frac{1}{\rho}P_d(x - X_2(t)),$$

2 wave makers

PHYSICAL REVIEW FLUIDS **2**, 053601 (2017)

Orbiting pairs of walking droplets: Dynamics and stability

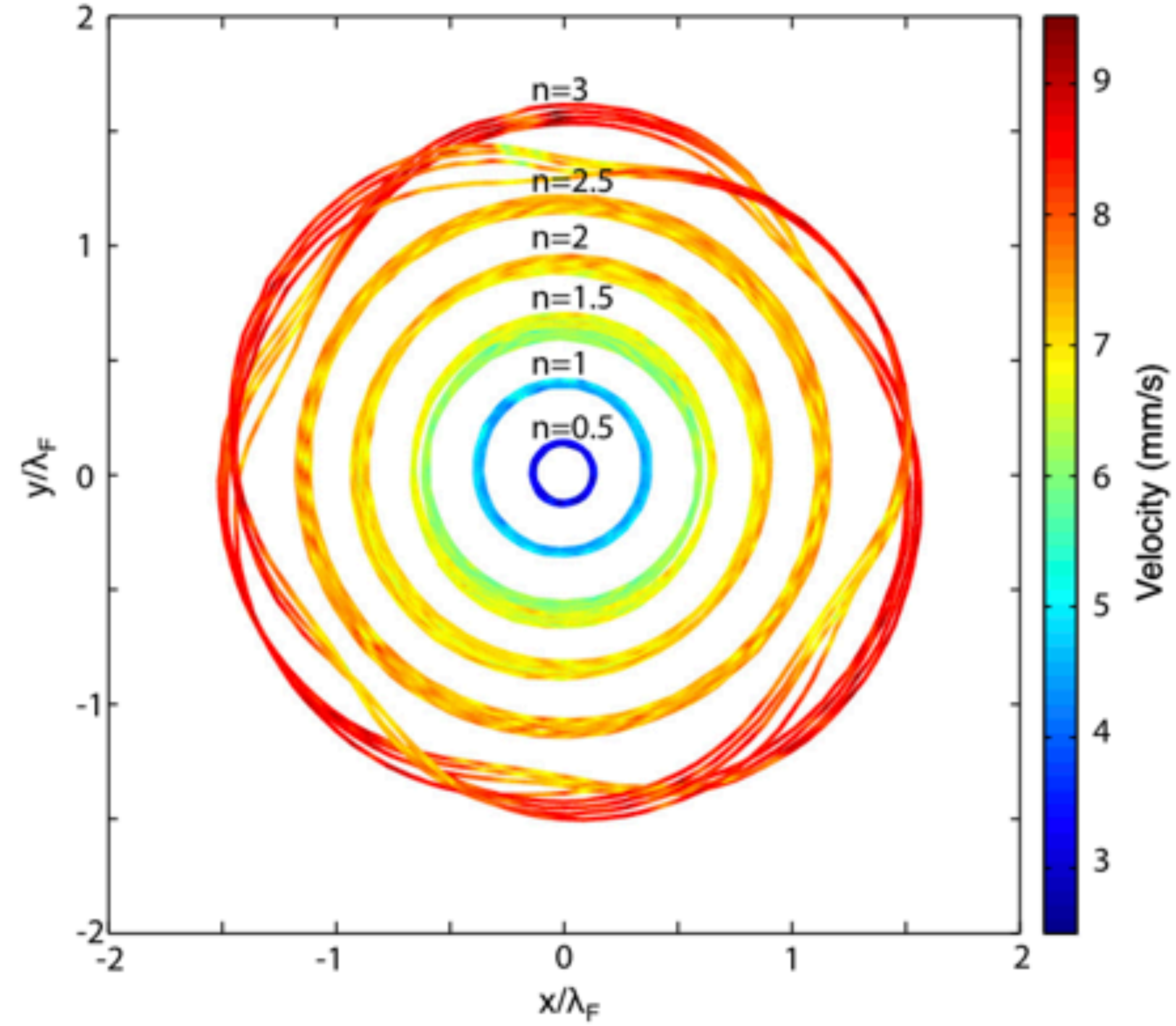
Anand U. Oza
Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012, USA

Emmanuel Siéfert
Department of Mathematics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA and Laboratoire PMMH, CNRS, ESPCI, PSL Research University, 10 rue Vauquelin, 75005 Paris, France and Sorbonne Universités, Université Paris Diderot

Daniel M. Harris
Department of Mathematics, University of North Carolina at Chapel Hill, Phillips Hall CB#3250, Chapel Hill, North Carolina 27599, USA

Jan Moláček
Max Planck Institute for Dynamics and Self-Organization, Am Faßberg 17, 37077 Göttingen, Germany

John W. M. Bush*
Department of Mathematics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA



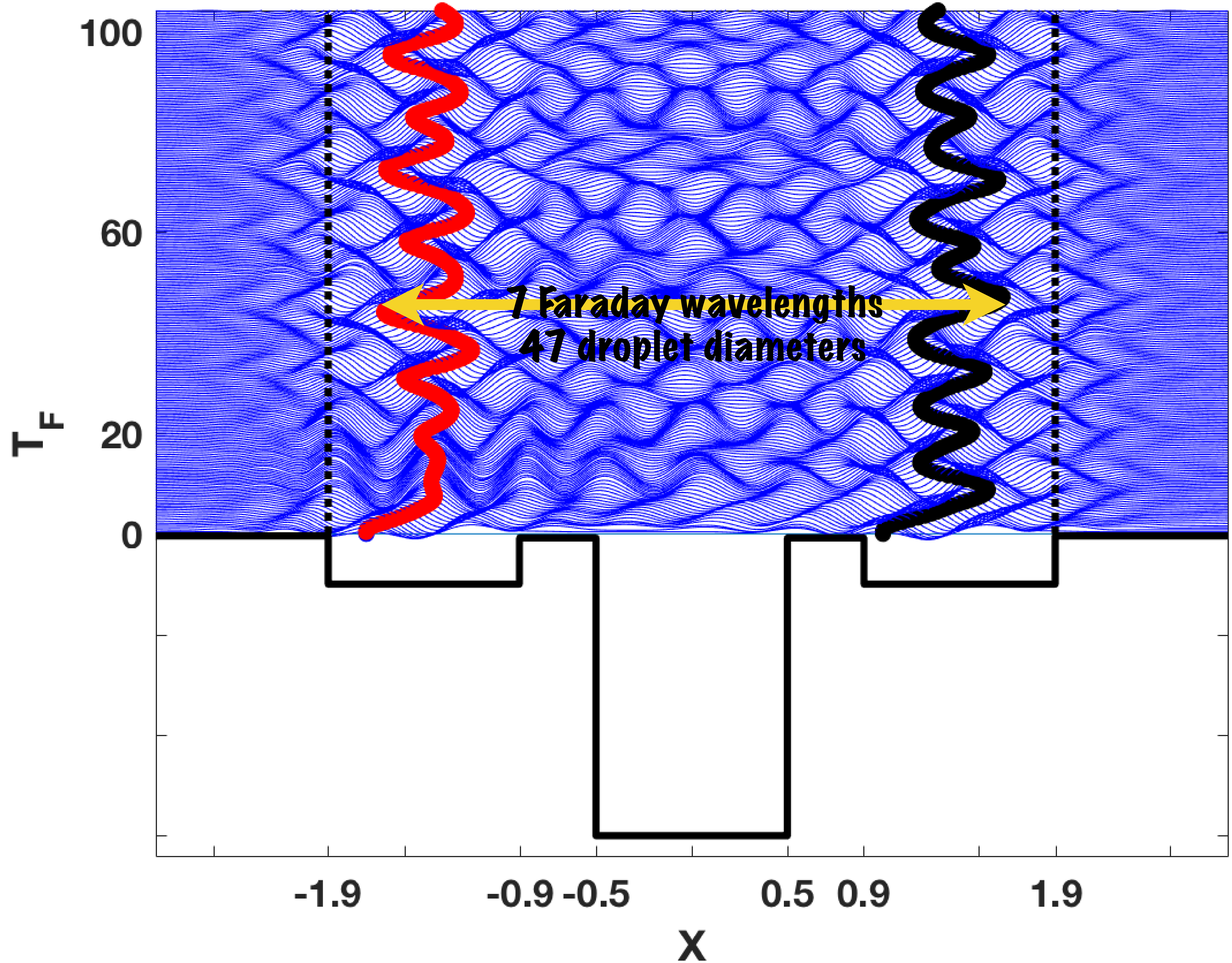
OUR GOAL:

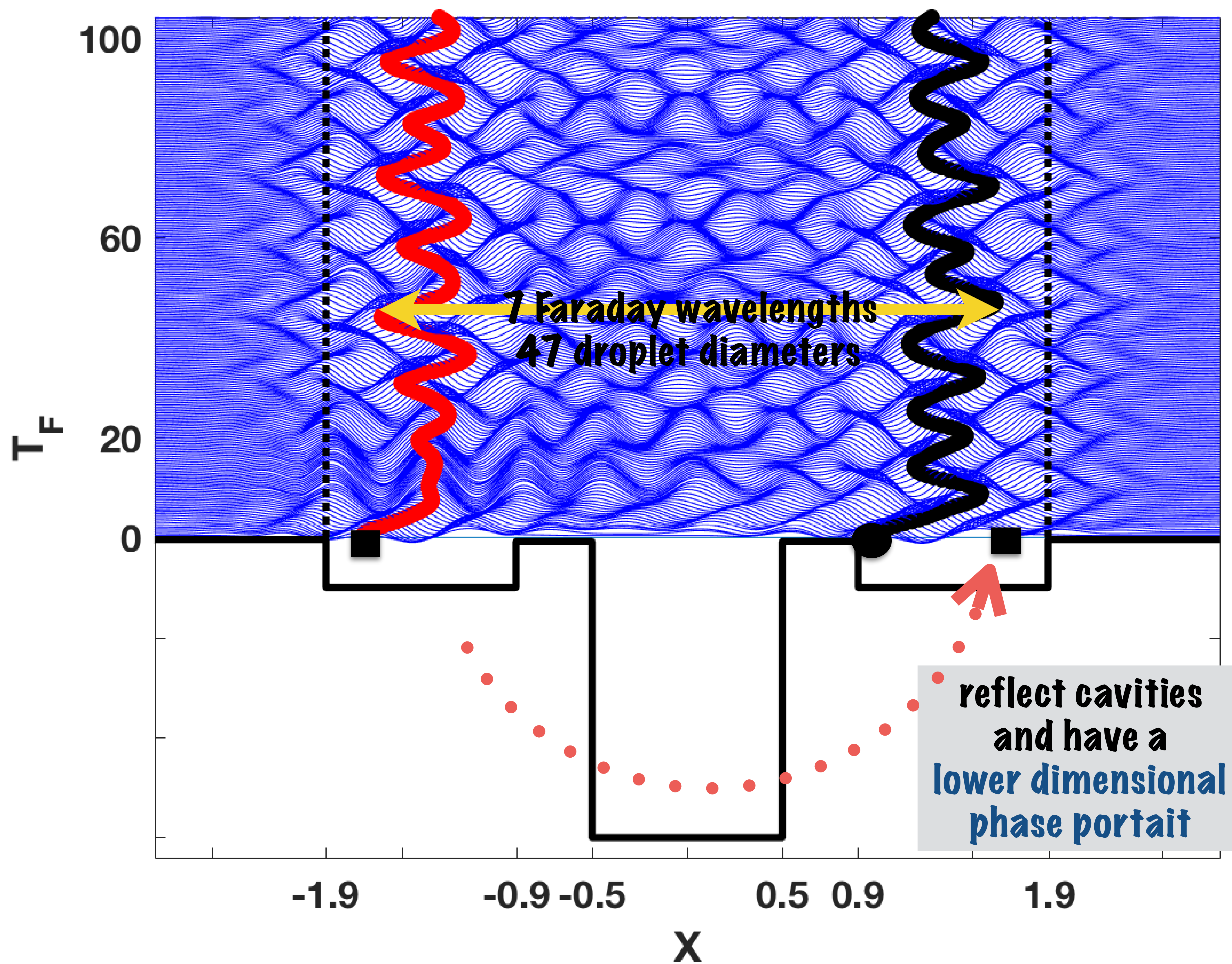
STUDY
TWO DROPLETS
(two oscillators)
set FAR APART

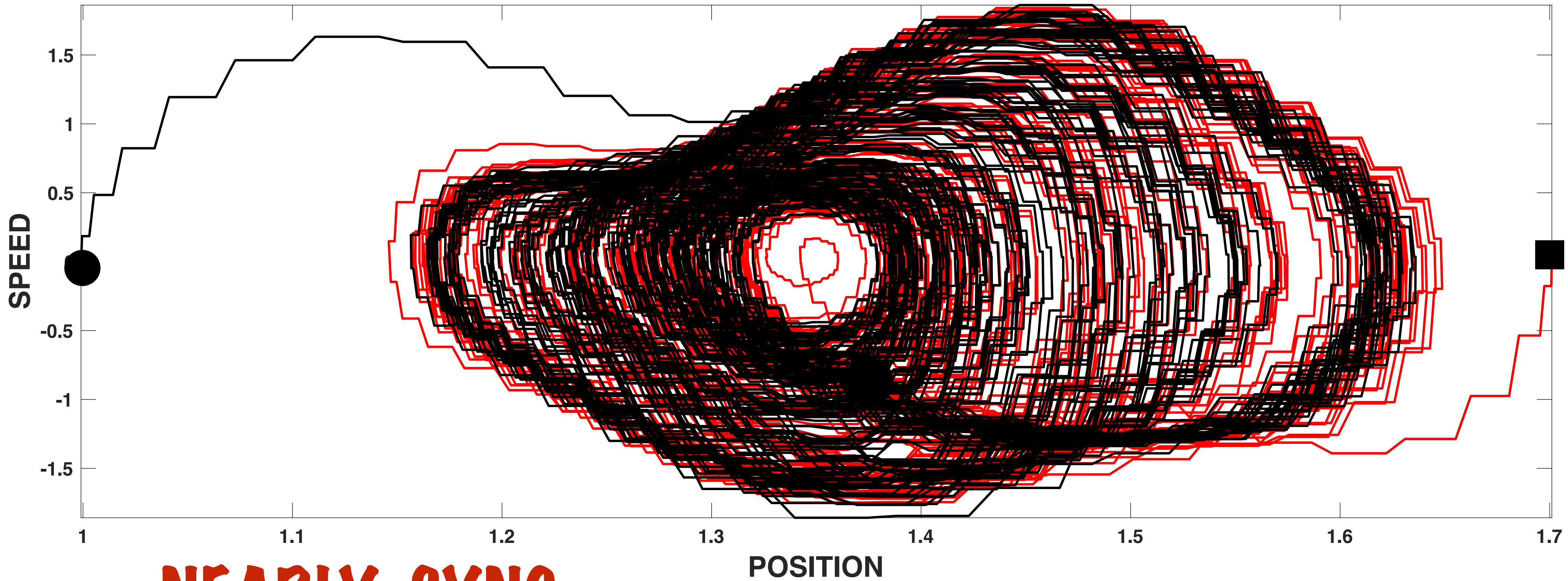
can they SYNC or be
CORRELATED
at a distance?

THREE CAVITIES

TWO DROPLETS

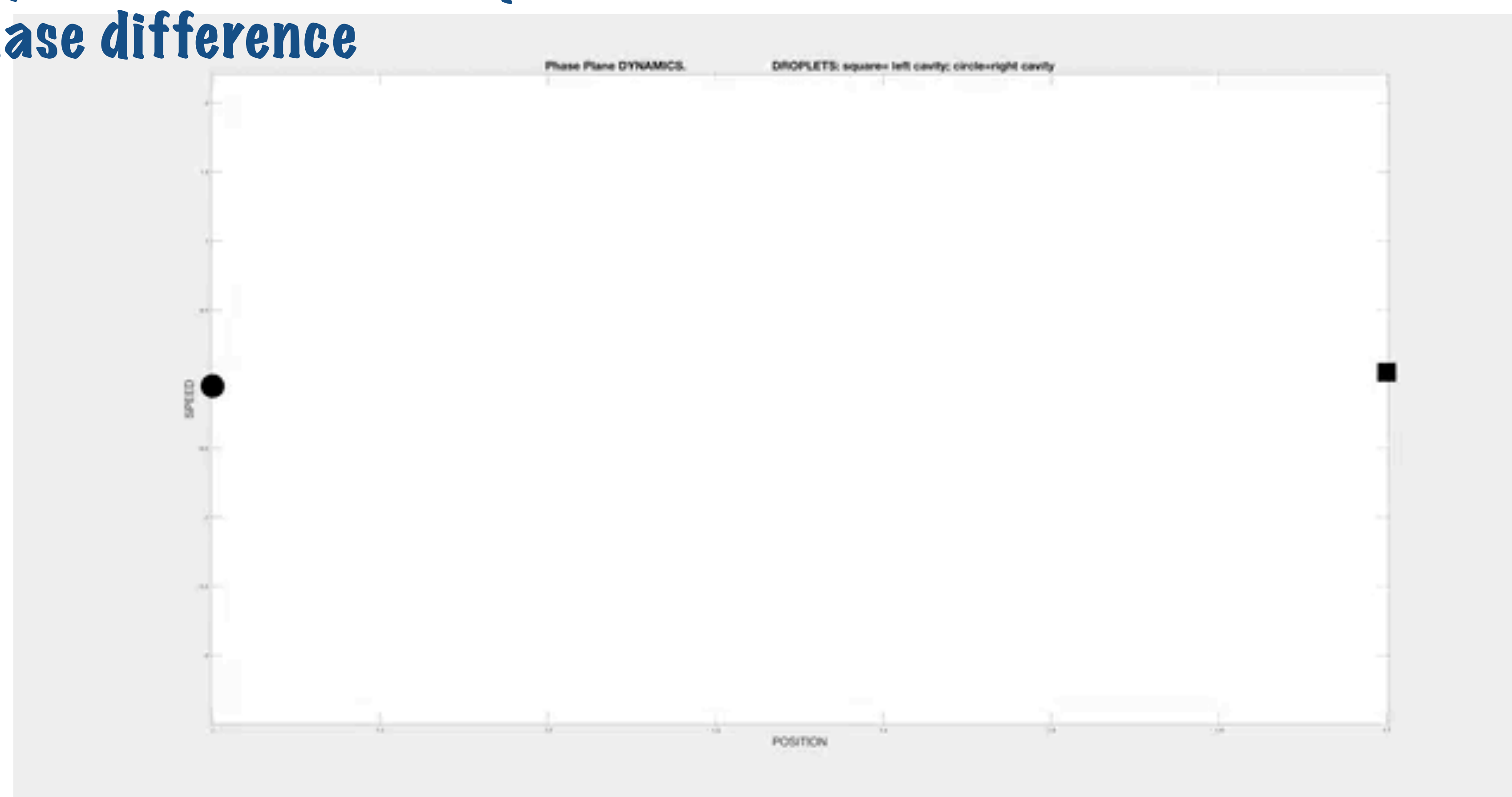

100
60
20
0
T_F
7 Faraday wavelengths
47 droplet diameters
-1.9
-0.9
-0.5
0.5
0.9
1.9
X


100
60
20
0
T_F
7 Faraday wavelengths
47 droplet diameters
reflect cavities
and have a
lower dimensional
phase portait
-1.9
-0.9 -0.5
0.5 0.9
1.9
X



NEARLY SYNC

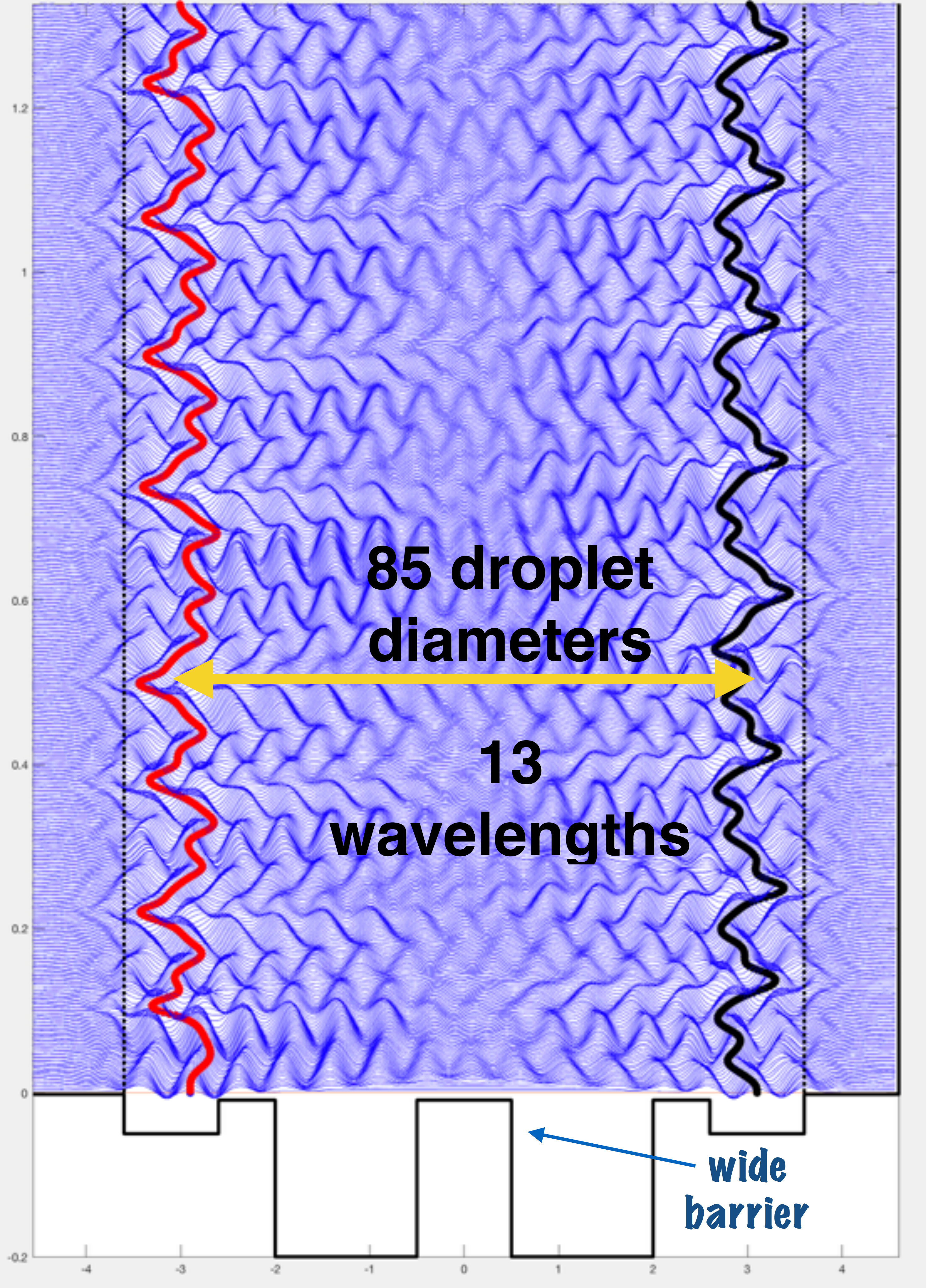
* having cycles of variable amplitude
* small phase difference



FOUR CAVITIES

TWO
DROPLETS

FAR APART

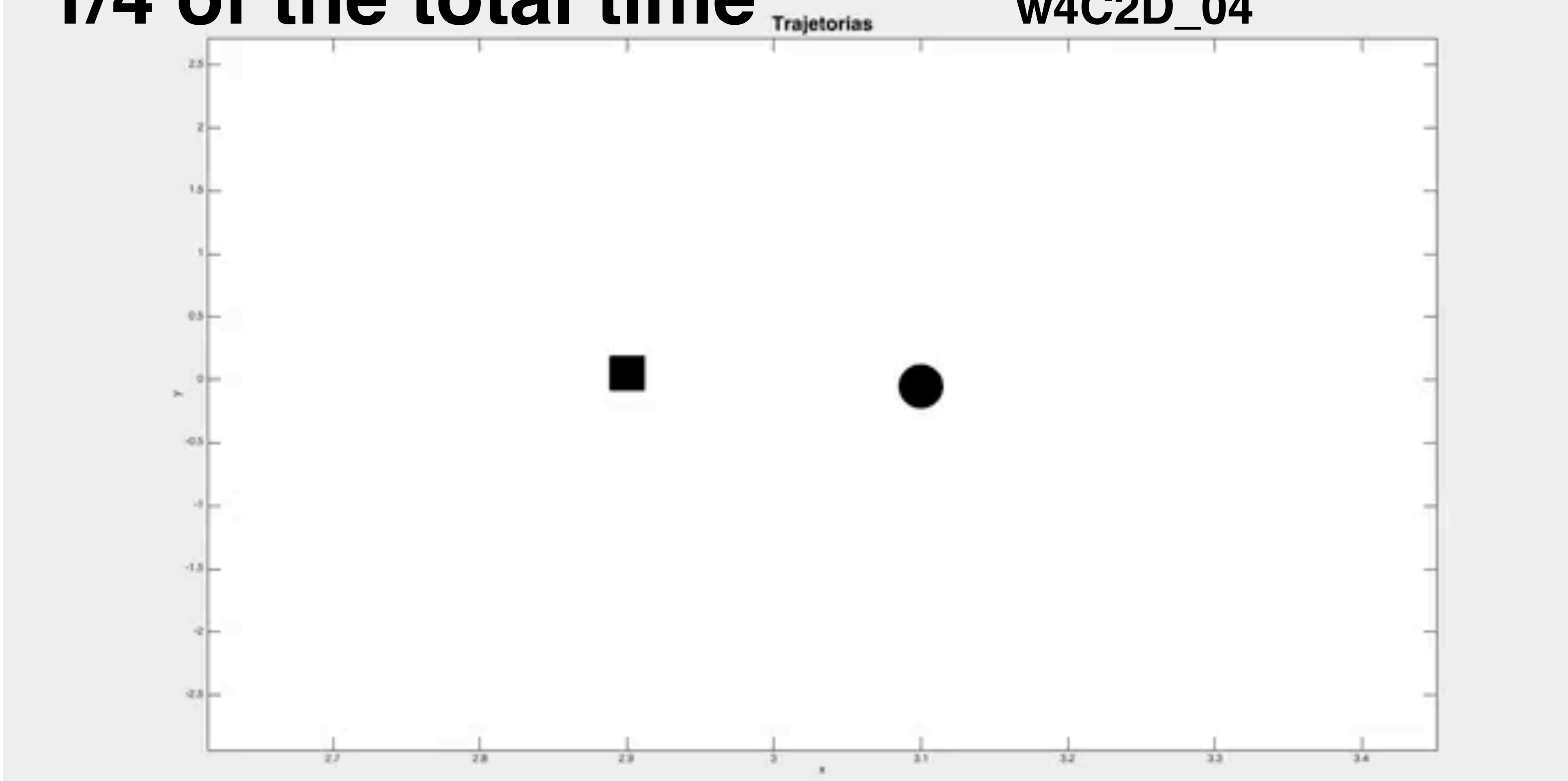


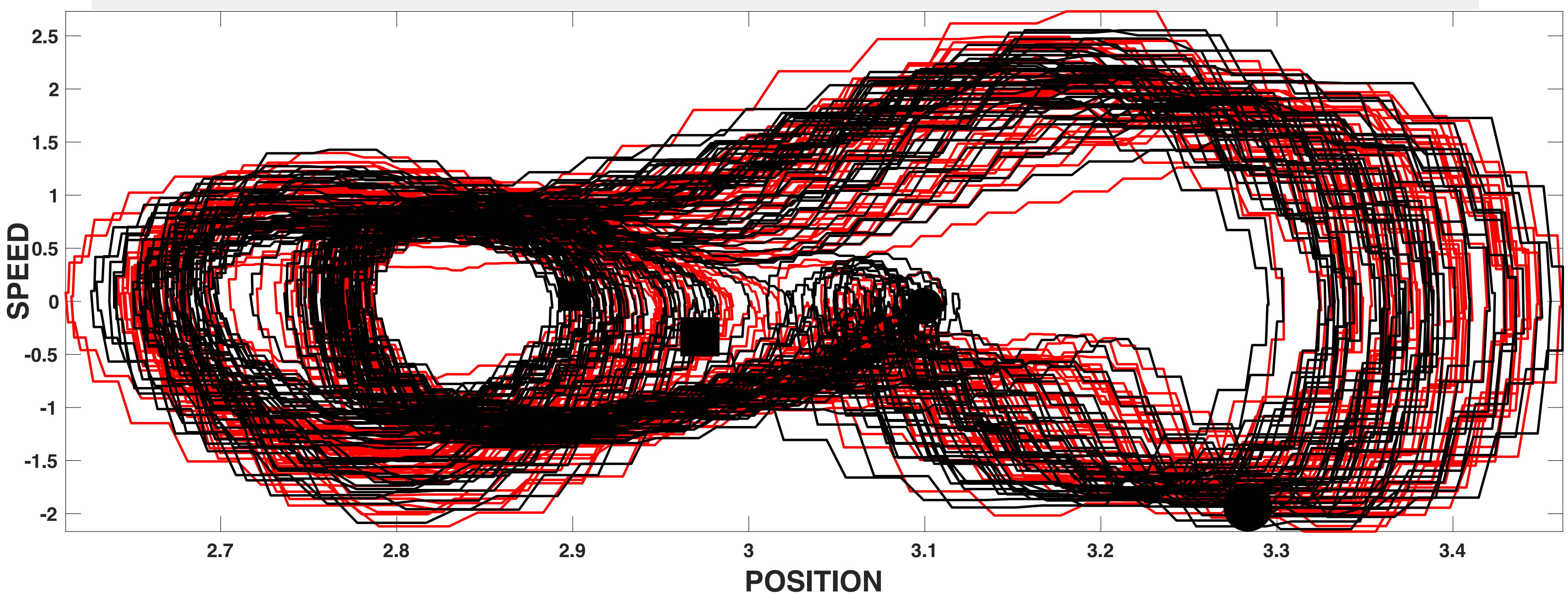
85 droplet diameters
13 wavelengths
wide barrier

Phase space animation

1/4 of the total time

w4C2D_04

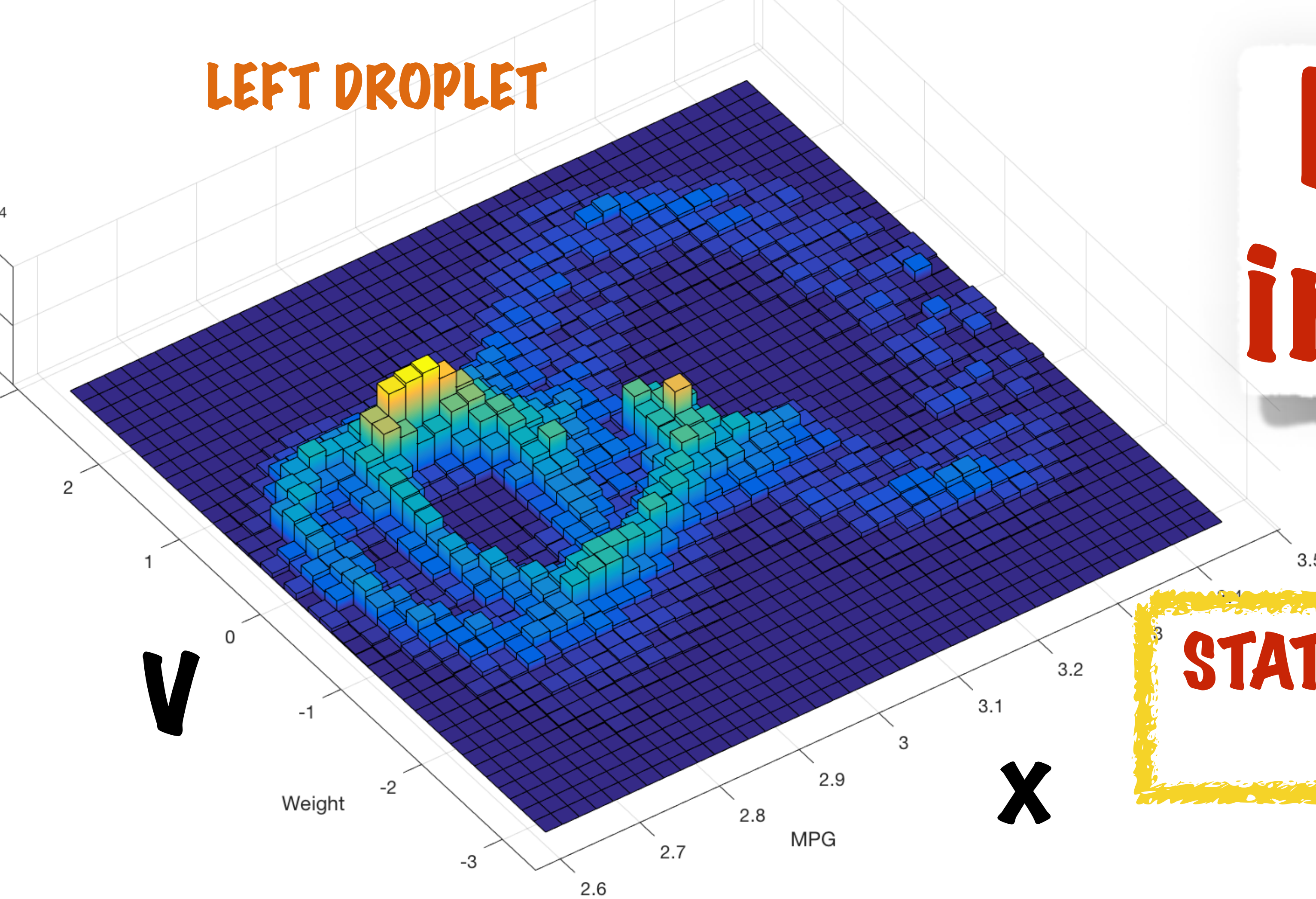


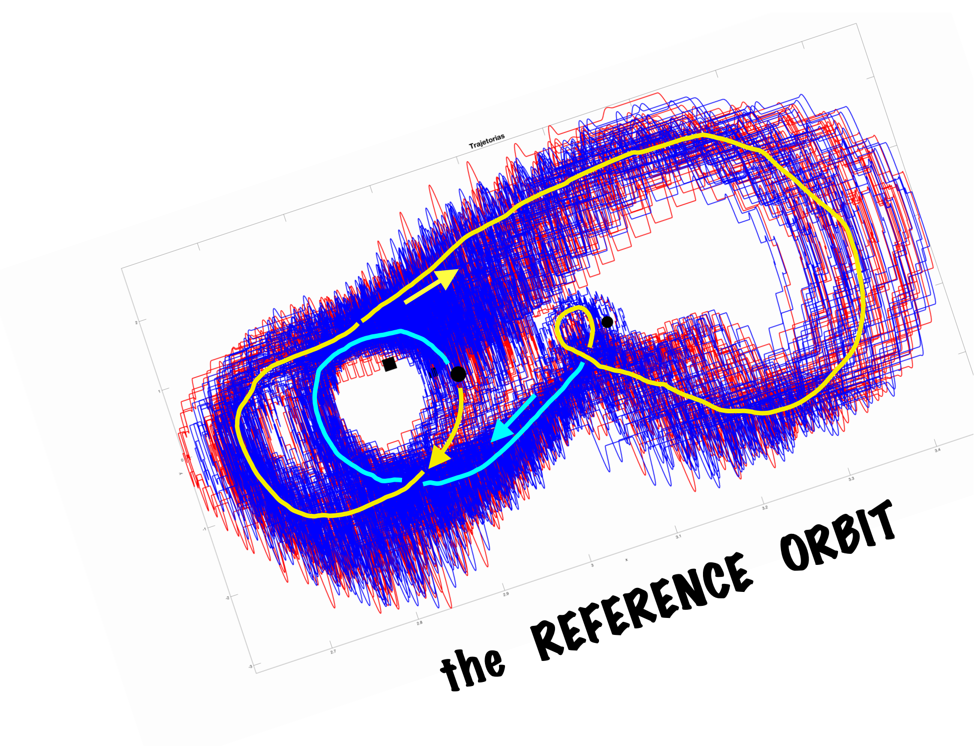


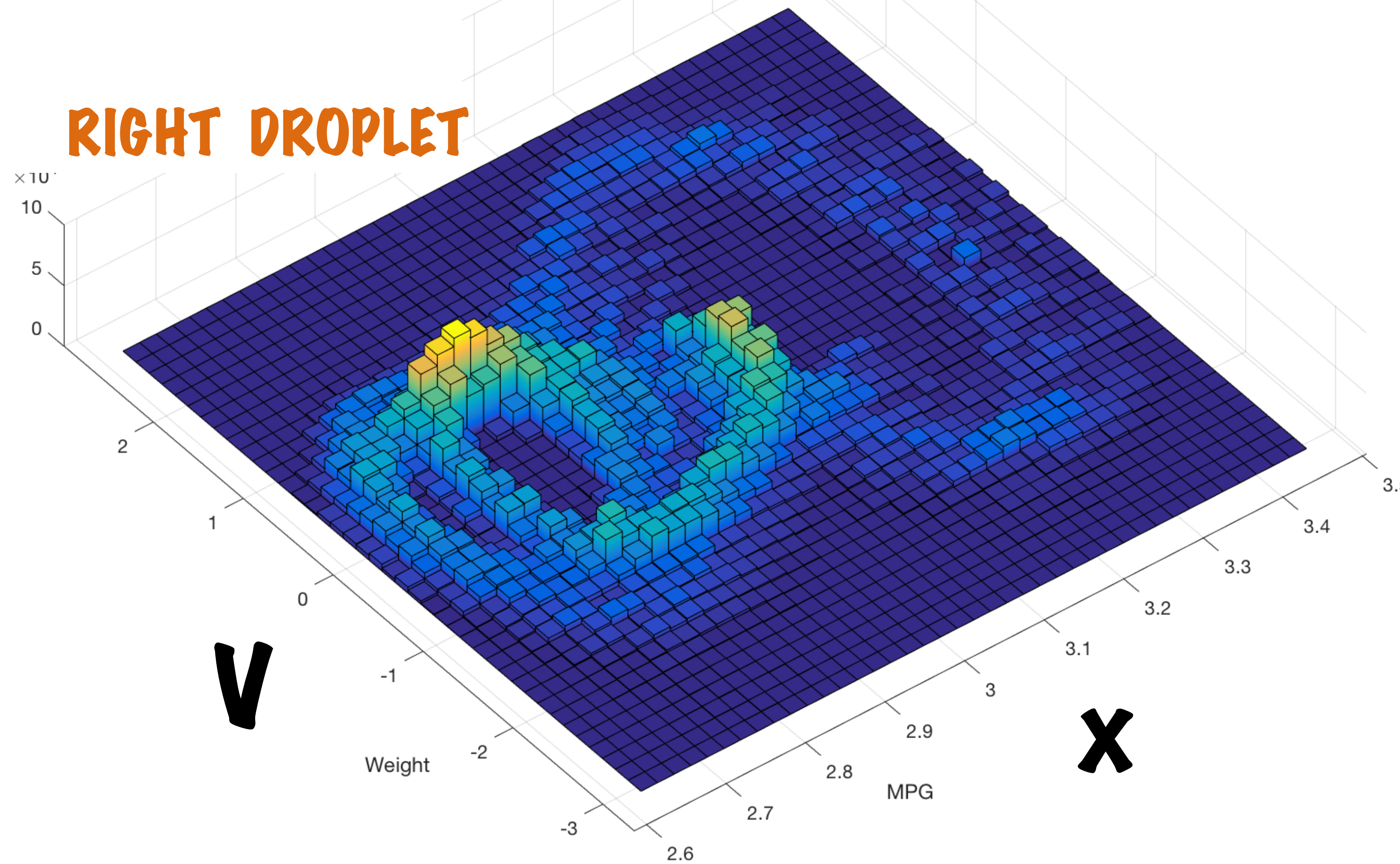
HISTOGRAMS in PHASE SPACE

STATISTICALLY INDISTINGUISHABLE

STATISTICAL COHERENCE

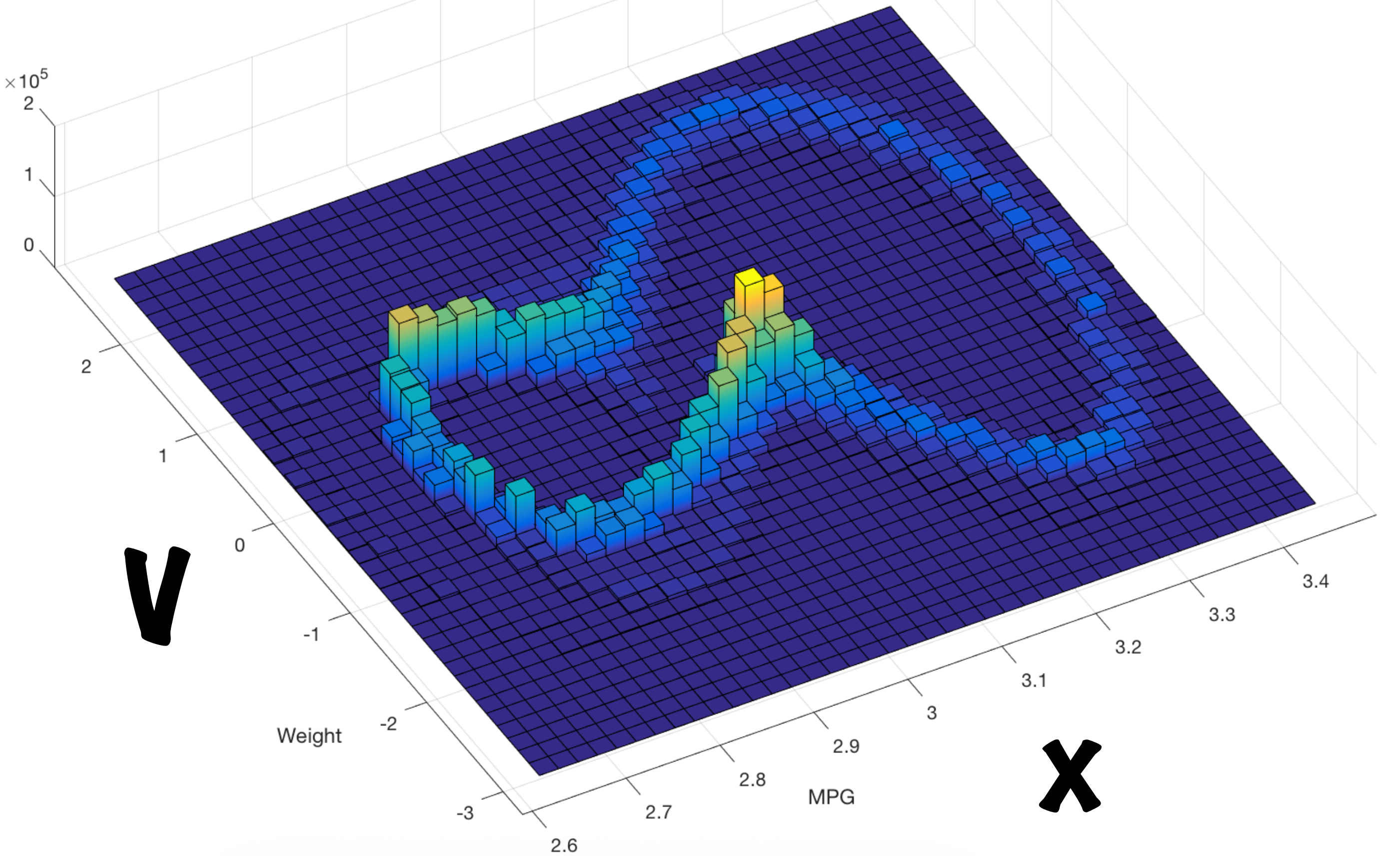




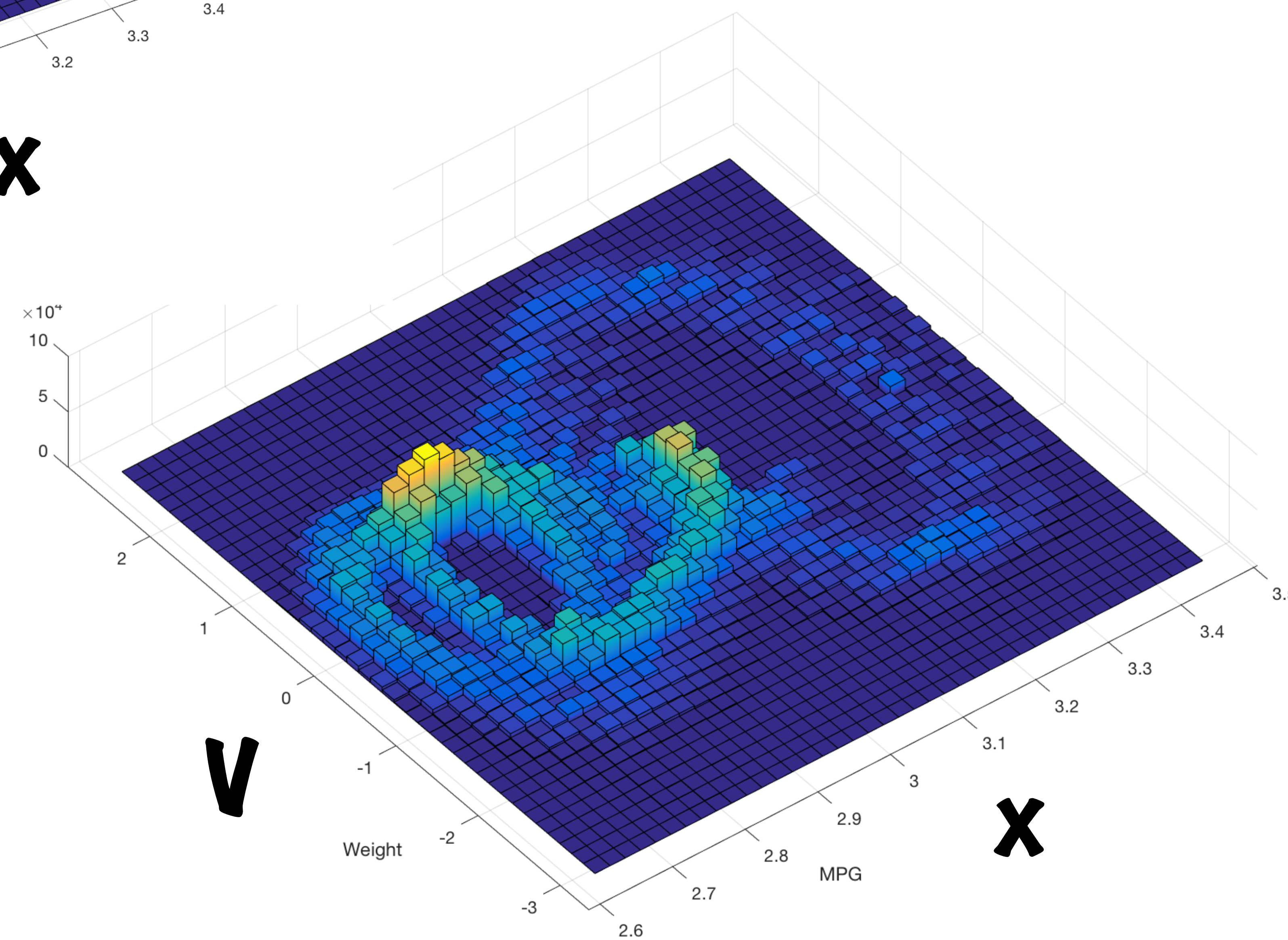


SINGLE DROPLET DYNAMICS

RIGHT DROPLET



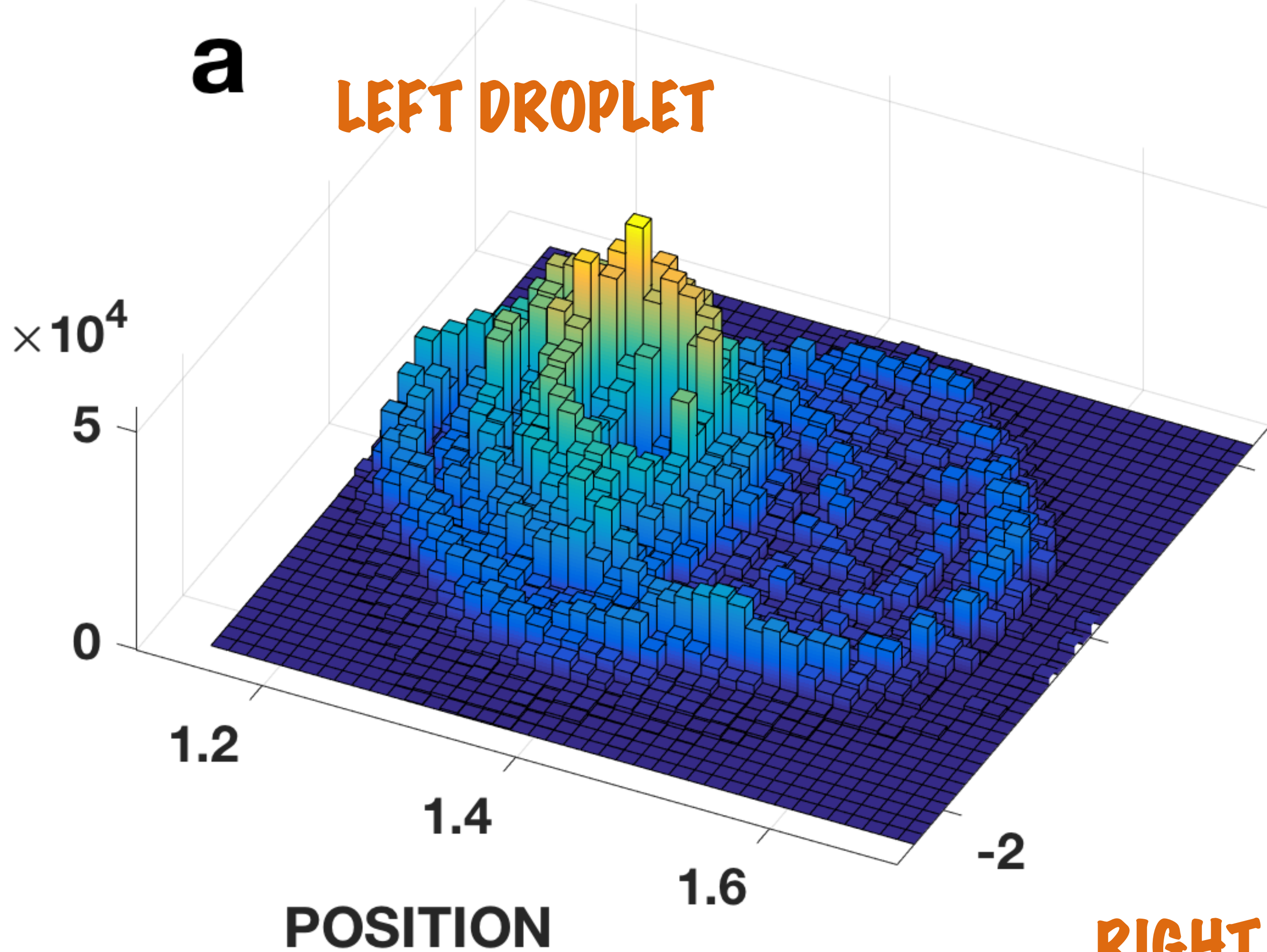
Phase space dynamics is described by the SYSTEM as a WHOLE and NOT by each particle independently

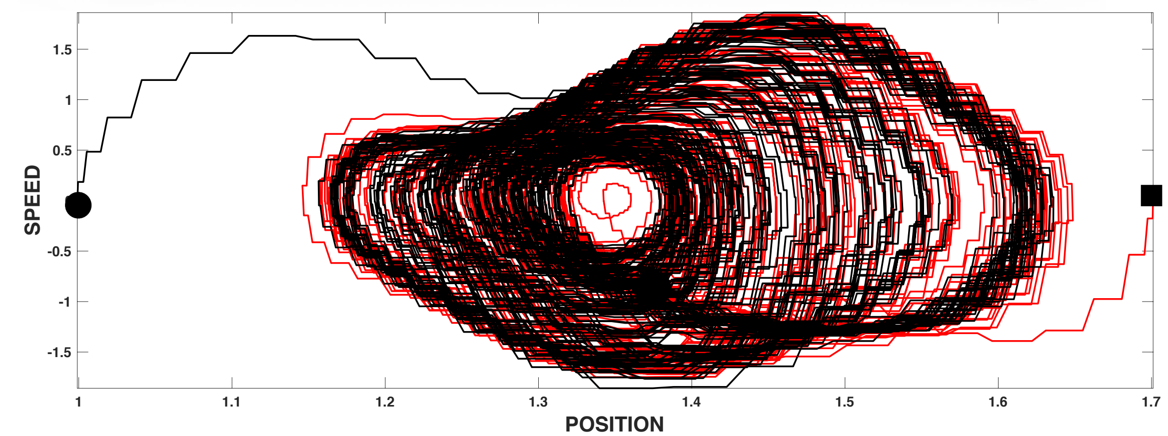


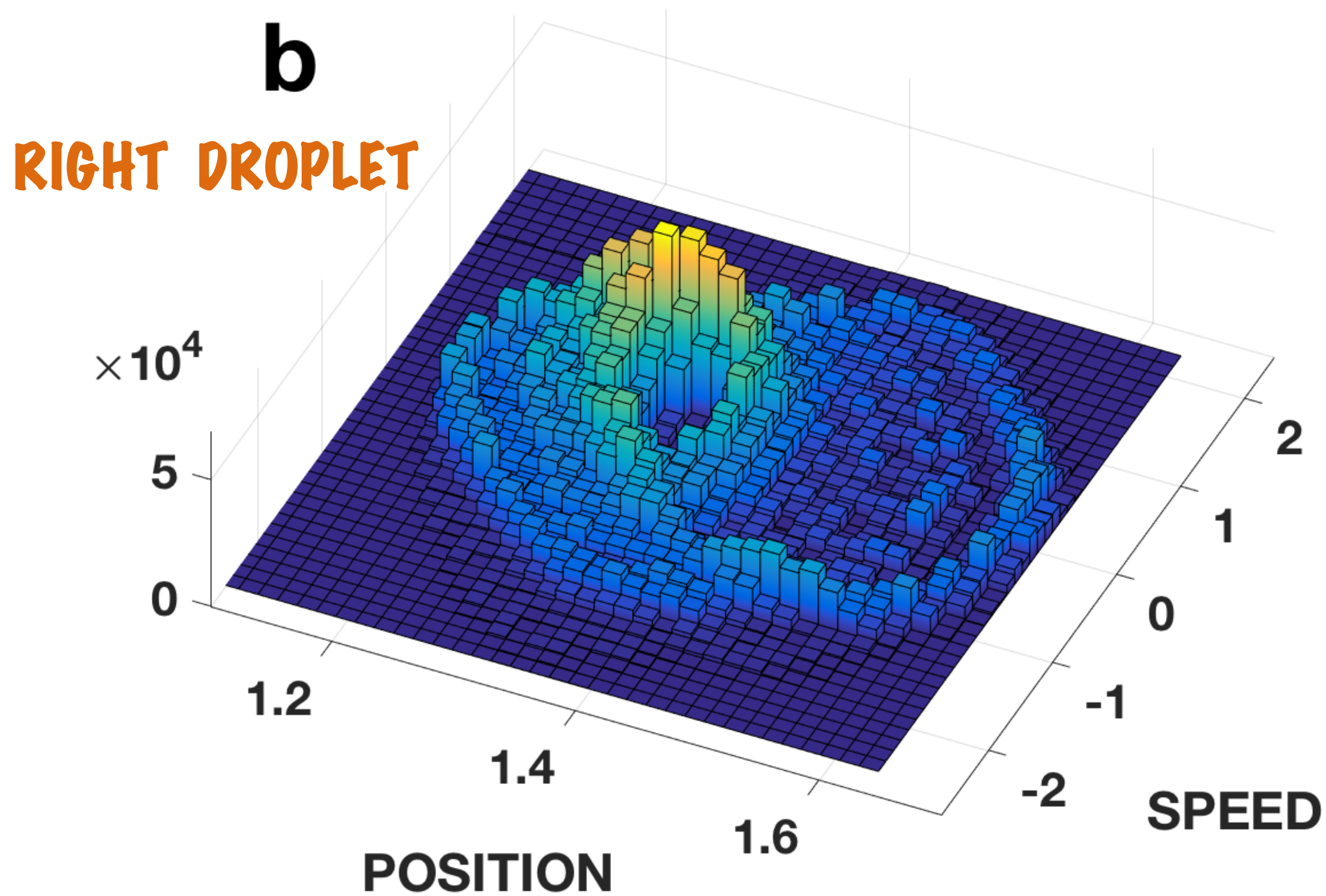
cannot be factorized

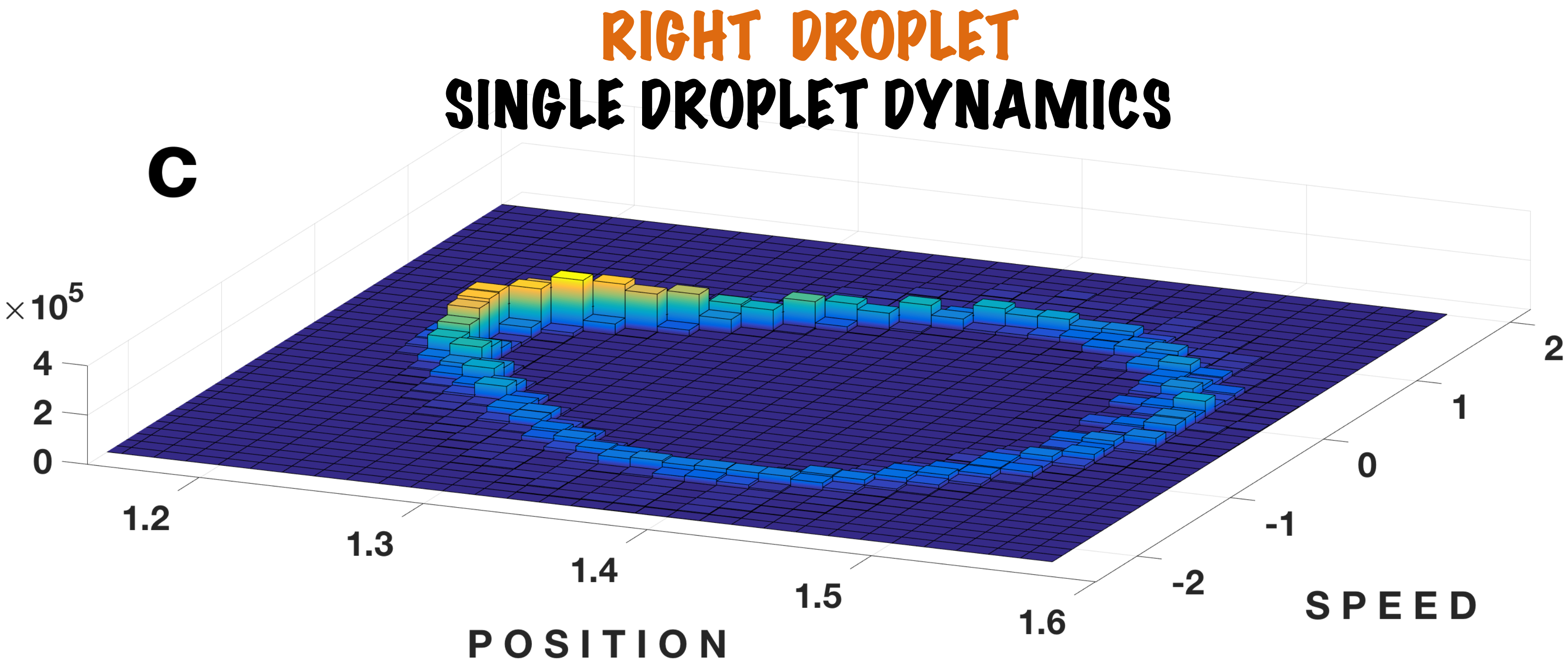
PHASE SPACE HISTOGRAM

STATISTICALLY INDISTINGUISHABLE particles

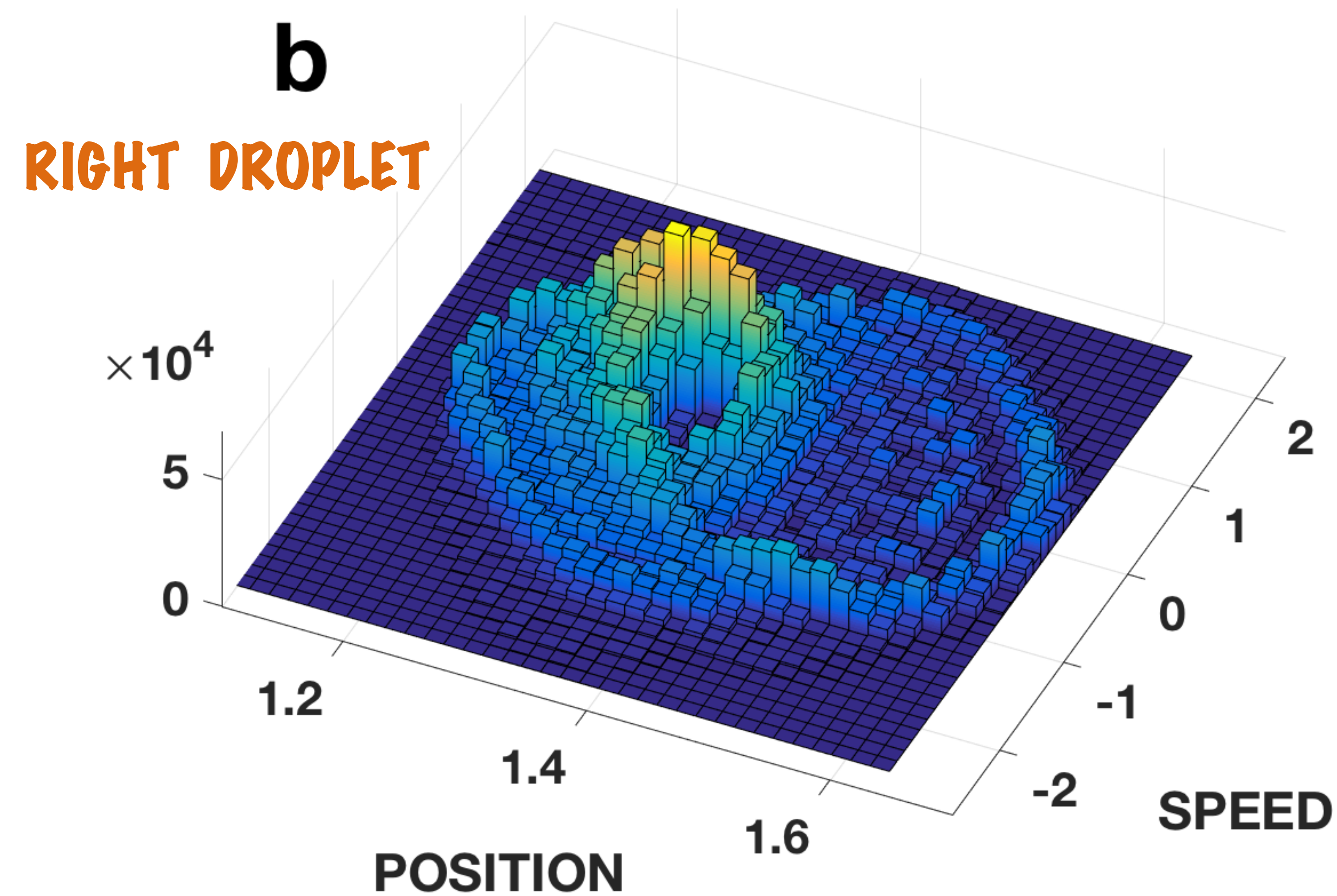


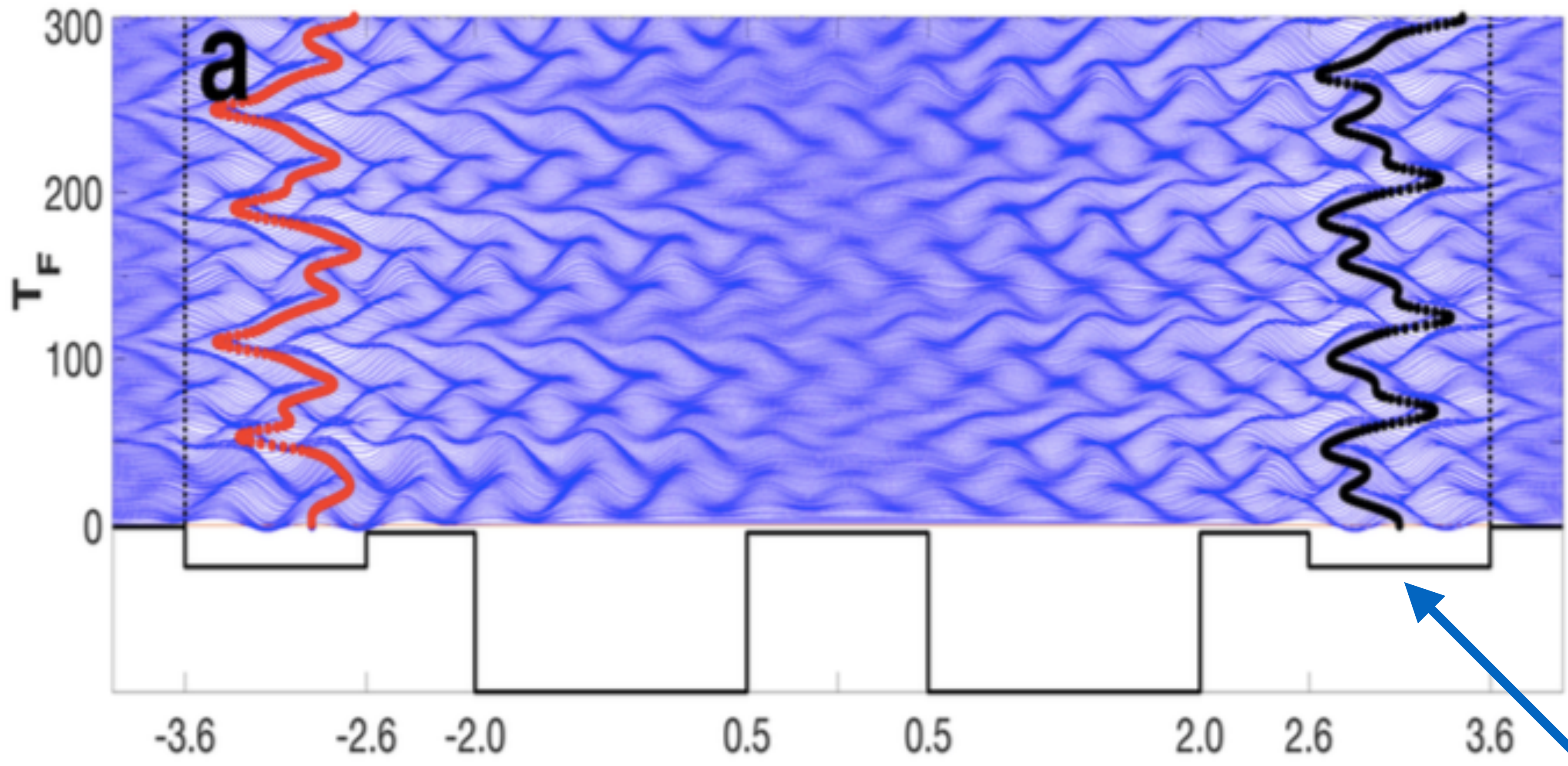






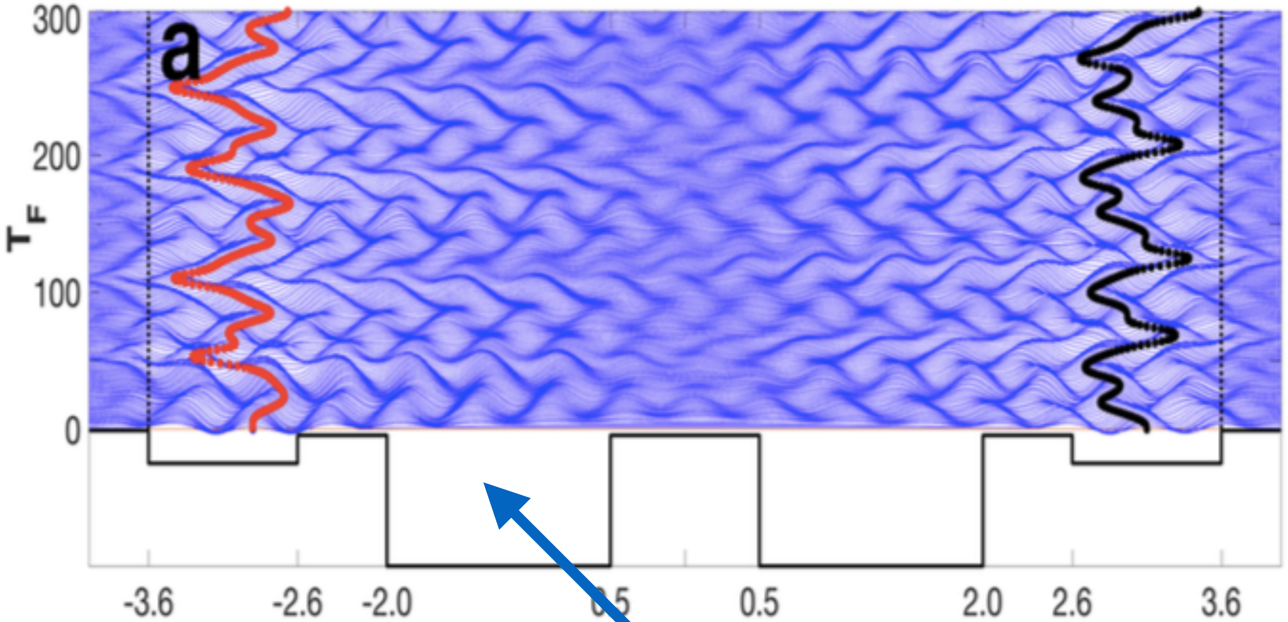
Phase space dynamics is described by the SYSTEM as a WHOLE and NOT by each particle independently





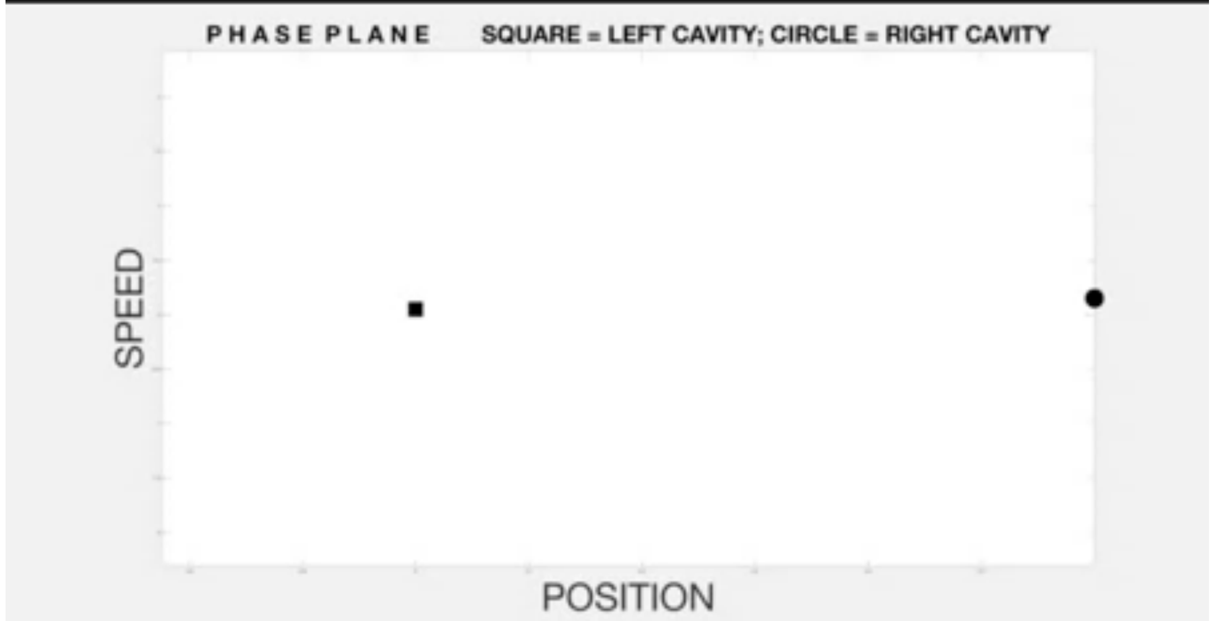
perturbed to 1.2cm
"different particle"

LOOSE statistical coherence



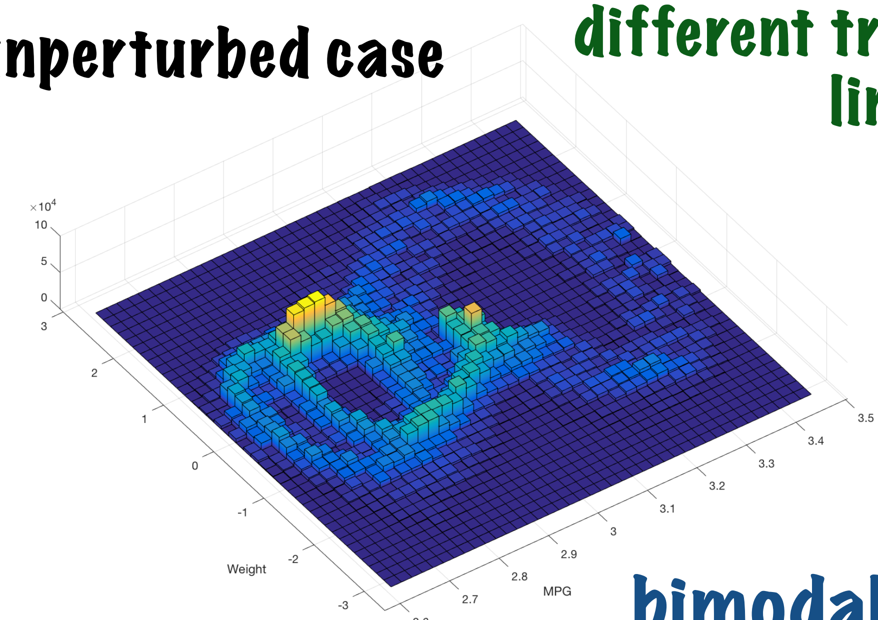
perturbed to 1.7cm

different transmission line

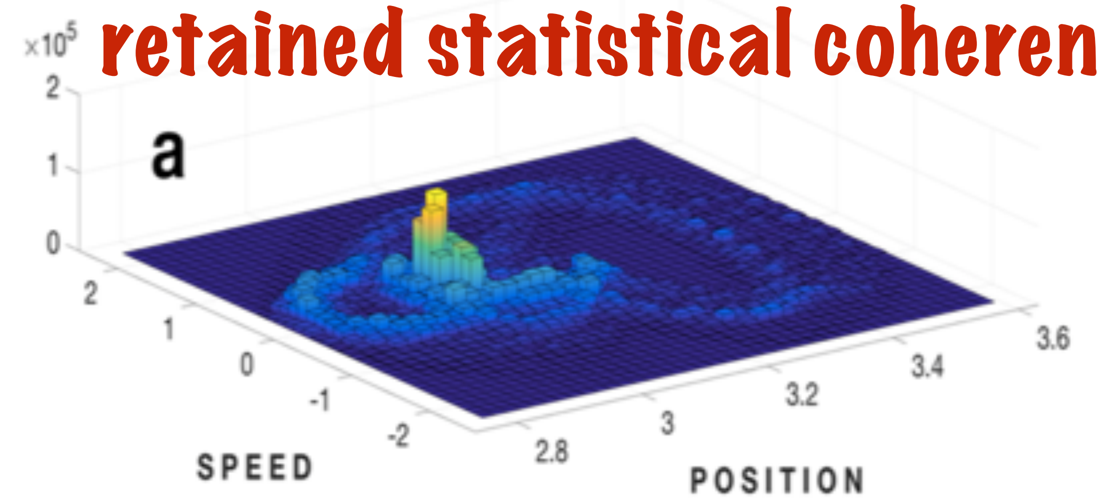


still statistically indistinguishable

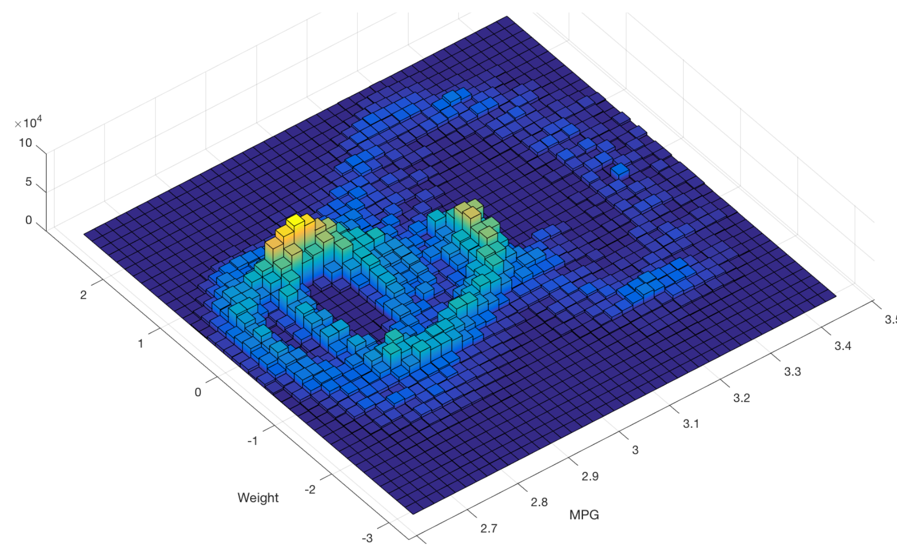
unperturbed case



retained statistical coherence

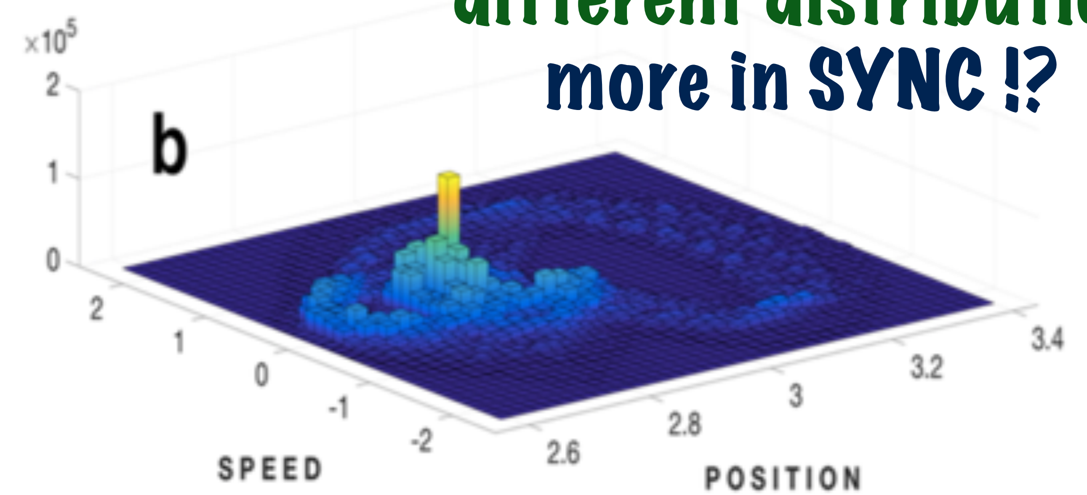


bimodal



different distribution

more in SYNC !?



We have shown a dynamical example reminiscent of entanglement

It has features of SED where the underlying ZPF plays a fundamental role in conveying information between the two uncoupled particles

Stochastic electrodynamics (**SED**) is an extension of the de Broglie–Bohm interpretation of quantum mechanics, with the electromagnetic zero-point field (ZPF) playing a central role as the guiding pilot-wave. The theory is a deterministic nonlocal hidden-variable theory.[2][3] It is distinct from other more mainstream interpretations of quantum mechanics such as the Copenhagen interpretation and Everett's

quantum state of each particle cannot be described independently of the state of the other(s), even when the particles are separated by a large distance—instead, a quantum state must be described for the system as a whole.

THANK YOU for YOUR ATTENTION !!

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