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Turbulence by Uriel Frisch, OCA, Nice, France . English version presented in Moscow, Beijing and Rio

The word "turbulence" originally meant "disordered movements of a crowd" (in Latin turba means crowd). In the Middle Ages in France "turbulence" was used as a synonym for "troubles". Thus, on an old French manuscript displayed at the J. Paul Getty Museum in Los Angeles, some years ago I found a "Seigneur, délivrez nous des turbulences" [English: "Lord, deliver us from turbulence"; Portuguese: "Senhor, livra nois das turbulencias"] As you can see, the meaning then evolved. First, turbulence is part of everyday's experience: no need for a microscope or a telescope to observe the volutes of the smoke of a cigarette, the graceful arabesques of the cream poured into coffee, or the entanglements of eddies in a mountain stream [Figure 1]. What we see is very complex, it's quite messy but it is very far from being total disorder. When we look at a turbulent flow, even in snapshot, on a photo, what we see is far more fascinating than the kind of total chaos obtained, for example, by projecting a handful of dry sand onto a sheet of paper. Turbulence, when you observe it, is full of structures, especially "eddies," entities known since Antiquity, studied and painted by Leonardo da Vinci (which was probably the first to use the word turbulence - turbolenza in Italian - to describe the complex motion of water or air). I believe it's this intimate mix of order and disorder that in fact constitutes both the charm and, it must be stated, one of the main difficulties of turbulence.

It is very easy to get turbulence. In fact, every time a fluid flows around a obstacle, for example in the wake of a boat, provides and the speed is is not too low, well, we'll have turbulence. So there is turbulence everywhere: the blood flow inside our blood vessels, the air flows around an automobile or a plane - responsible for the famous "turbulence" for which we are asked to attach our seat belts - or the motion of the atmosphere in meteorology, the motion of gas inside stars such as our Sun, and finally the density fluctuations of the early Universe giving rise later to the great structures of the present Universe, such as the clusters of galaxies [Figure 2]. Without all this turbulence, urban pollution would persist for millennia, the heat produced by the nuclear reactions in the stars would not be able to escape on an acceptable time scale and the weather would become predictable over very extended periods.

The equations governing the movements of fluids, whether turbulent or not, have been written for the first time by Claude Navier in 1823. They are often called Navier-Stokes equations because of improvements made later by George Stokes. In fact it is essentially Newton's equations, which connect the force and the acceleration, equations that must be applied to each parcel of the fluid what was done for the first time by Leonardo Euler in the middle of the eighteens century. Navier's crucial contribution was to add to the Euler equations a friction term between the various fluid layers proportional to coefficient of viscosity and speed variations [Figure 3]. These equations, which can be solved with powerful computers, still involve major challenges to which I shall come back.

Turbulence became an experimental science towards the end of the 19th century when Englishman Osborne Reynolds was able to observe the transition from laminar to turbulent regime. You know that in a pipe, if the water flows slowly, the flow will also be very regular. This is called a laminar flow. If the flow is fast enough, we observe the formation of turbulence with many eddies. Reynolds discovered rather simple laws applying to any pipe for this transition to turbulence; he introduced a number, called since the Reynolds number, which is simply the product of the diameter of the pipe D and the average velocity of the flow in the pipe V, divided by the (kinematic) viscosity of the fluid nu (viscosity of the air approximately 0.1 cm 2 / S, viscosity of the water 0.01 cm 2 / S) is R = DV / nu. Reynolds has shown that when this number exceeds a certain critical value, the order of a few thousands, then all of sudden, the flow becomes turbulent. Transitions similar but more spectacular are observed in open flows behind a cylinder [figure 4]. Leonardo had already seen the phenomenon of eddy street and had is almost correct [Figure 5].

A very important characteristic of these turbulent flows, which appears at the transition, is their chaotic character. More precisely, turbulent flows appear as unpredictable. What does it mean, non-predictable? Suppose we know in detail the configuration of the flow at a given moment. Although the flow is governed by well-defined deterministic equations, as we say, in practice, it is not possible to predict the subsequent evolution for long periods. This theory of chaos, which owes much to Henri Poincaré, David Ruelle, Edward Lorenz and to the Russian School of Kolmogorov and his pupils Vladimir Arnold and Yacov Sinai, has very important implications in meteorology. Let's imagine that, to predict time, we measure, at a given moment, the wind, the pressure, the temperature in all points of the planet and that we try to predict the subsequent evolution of time by computer calculation. In fact, after relatively short time, you will not be able to predict in a detailed way find the atmosphere, and this regardless of the power of computers. It is said that the atmospheric turbulence is unpredictable, it ends up being sensitive to the slightest sneeze or a flutter of a butterfly, as suggested by the American meteorologist Ed Lorenz. This "butterfly effect" is illustrated in Figure 6 where the curves represent not the trajectory of a butterfly but - symbolically - the trajectory of the point representative of the whole of the system studied. The upper black curve corresponds to the case without butterfly and the lower black curve to the trajectory modified by the initial presence of a wing flap of a butterfly. Both trajectories remain close at first (to show it I repeated a dotted version of the upper trajectory) then move off rather quickly. In practice it is not possible to predict in detail the weather will do beyond about ten days, although the situation may be more favorable in the tropics.

In geophysics and astrophysics gigantic Reynolds numbers of hundreds of millions and well beyond are commonplace. A very interesting point is that when increases the number of Reynolds, which can be done for example by reducing its viscosity, there appear more and more small eddies as you see in Figure 7 which presents a turbulent jet. Each vortex is a bit like a kind of molecule. It is what we call "degrees of freedom". So high Reynolds numbers mean that there are many degrees of freedom; this is called the fully developed turbulence regime. It is easy to observe this regime in a large wind tunnel like those where we test models of cars and planes. If we examines the behavior as a function of time of the speed at a point of such a flow measured by a probe, we are struck by the analogy with the Brownian motion curve [Figure 8]. The latter can be imagined by recording as a function of time the location of a drunkard walking the street of a village with many taverns; of course, our drunkard will sometimes move down the street and sometimes up the street, without ever remembering the previous direction. In other words, the drunkard performs a random walk. It's easy to see that the typical displacement of such a drunkard during a certain time interval is proportional, not to the elapsed time, but to its square root (the same law that governs errors in opinion polls). In fully developed turbulence one finds that the variation of speed during a certain time interval is proportional, not to the square root,but to the cubic root of time elapsed. This cubic root law, obtained in fact by a dimensional argument related to the conservation of energy, was predicted in 1941 by the Russian mathematician Andrei Kolmogorov and has been fairly widely validated by computer experiments and simulations. In fact, in 1922 the Englishman Lewis Fry Richardson, had sensed what was happening by presenting his vision of the cascade of energy from large to small scales of turbulent flow, vision directly inspired by a poem of the English poet Jonathan Swift:

Rather than trying to translate, I ask you to imagine a big flea in the process of sucking your dog's blood, blood that will here play the role that kinetic energy plays in turbulence. Now imagine that the big flea is in turn beset with more little fleas that suck its blood and so on until reaching fleas so small that the blood is broken down by molecular processes. It is clear that the monster thus come out of Swift's imagination is what Benoît Mandelbrot called a fractal. These fractals can be characterized by a dimension that is not an integer. The objects of whole dimension 0, 1, 2, 3 are, for example, points, lines, surfaces and volumes. To imagine an object of fractal dimension between 2 and 3 think for example of a cauliflower. The fractal dimension of turbulence - more precisely what mathematicians call the Hausdorff dimension of energy dissipation - is very close to three. Would it be exactly three, the theory proposed by Kolmogorov in 1941 would be correct, which explains the success of this theory in developing empirical models for computations engineers. The calculation of such dimensions from the fundamental equations of the mechanics of fluids remains an open problem. However, important progress has been made in the last 20 years using mathematical tools borrowed from quantum field theory, applied to a simplified model due to the American Robert Kraichnan. In this model one assumes that turbulent flow is prescribed and one seeks to characterize the properties of a tracer transported by this turbulence, as illustrated by figure 9 of Antonio Celani, Alain Noullez and Massimo Vergassola, representing a snapshot of the concentration of a tracer obtained by simulation the computer. One can imagine for example that it is the concentration of a released pollutant in the ocean, We now know how to calculate the fractal properties of such pollutants, but it will probably years before being able to carry out a comparable business for Fractal properties of turbulence itself.

In a turbulent flow, the temporal variation of the velocity at a point is generally not very far from being given by cubic root law of Kolmogorov, but we have known for a long time that things can be more complex. Already in 1843 Adhémar Barré de Saint Venant observed that "the flow in the channels of large sections, those of which we would say today that they have a large Reynolds number, present ruptures, eddies and other complicated motions. "The interesting point are the ruptures. It is found experimentally that the velocity may occasionally, vary considerably between two neighboring points. If by chance the scale of this variation becomes comparable to the distance traveled by the molecules of the fluid between two successive collisions, then we need to rethink the mathematical foundations of the Navier-Stokes equations. The traditional way of obtaining these equations supposes indeed a strong separation between the microscopic world of molecules and the world, called "macroscopic" where the fluid is treated as a continuous medium.

This brings me to the great mathematical challenge that is the subject of one of seven awards in the amount of one million dollars announced by the Clay Foundation in the year 2000. The problem is to show that the Navier-Stokes equations lead to a well-posed problem. This means that if we know the motion of the fluid at an initial moment we are able to show that there is a unique solution at any later time. Note that in the present case our problem is not about error growth in time but about the uniqueness of the solution. This problem has been solved in the thirties by Jean Leray in the case of two dimensions of space (which is relevant in meteorology and oceanography). The problem is much more difficult in three dimensions. I will try now to give a very small glimpse of the difficulty without use of advanced mathematical formalism. First of all it should be noted that in a fluid that is not in uniform motion the fluid threads rub against each other, because of the viscosity, which tends to slow down their relative movement. At low speed, so at low numbers Reynolds (the latter is proportional to the speed), the effect of viscous friction is very important for all the eddies present in the flow. This friction flattens everything and we can prove - it is not very difficult - that the problem is well posed. In contrast, at large Reynolds numbers, the effect of viscous friction is limited to the smaller vortices and the problem is close to the problem of an ideal (inviscid) fluid in which the viscosity is ignored. We know that the latter problem is well posed for a short time but not beyond. Roughly, the best we know, for now, is that the ideal flow does not behave better than a mobile object whose acceleration would be proportional to the square of its speed, an assumption that leads to a catastrophic increase in the speed that becomes infinite in a finite time; in other words it blows up [Figure 10]. Some computer numerical simulations suggest that the ideal fluid is actually much wiser, does not explode, and thus leads to a problem well posed for arbitrarily long times. It is also possible that the ideal fluid does blow up but that the effect of viscous friction prevents this. This is precisely what happens in Kolmogorov's 1941 theory, but not necessarily so in reality.

To conclude, I would like to emphasize that turbulence has a very special status in contemporary physics. It is often considered as one of the big open issues of physics, but unlike other frontier problems of physics, the phenomena in which one is interested in turbulence are neither in the infinitely small nor in general, in the infinitely big. These phenomena are perfectly well described by Newtonian mechanics, without any need to have recourse to quantum physics or to relativistic mechanics, that is to the modern ideas of physics on space, time and matter. As you can see, "classical" physics, i.e., the one taught in high school, still has some great mysteries.