IMPA Summer Course Syllabus.
An introduction to stochastic differential equations with applications to fluid dynamics

Instructors.

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Course Description. This course will provide a practical introduction to stochastic differential equations, with a focus on applications in fluid dynamics. Specific topics include: basic probability, statistical dynamics (Itô calculus, moment hierarchies, Liouville/forward equations, path-integral methods) and basic fluid dynamics and turbulence theory. The Kraichnan model for turbulent advection of a passive scalar will be discussed in depth.

Pre-Requisites There are no formal prerequisites but an undergraduate courses in differential equations is very highly recommended. Some familiarity with a coding language such as MATLAB or python is also recommended. Concerned students should discuss their preparation with the instructors.

Textbook The primary source for the course will be lecture notes, which will be distributed. In addition, there may be cited journal articles. Recommended, but not required, texts are:

Cardy, J., Falkovich, G., and Gawędzki, K. (2008). *Non-equilibrium statistical mechanics and turbulence* (Vol. 355). Cambridge University Press.

Øksendal, Bernt. *Stochastic differential equations*. Springer, Berlin, Heidelberg, 2003. 65-84.

Risken, Hannes. The Fokker-Planck Equation. Springer, Berlin, Heidelberg, 1996. 63-95.

Grading/Class Project. There will not be homework or examinations in the course. Instead, class time on Fridays will be devoted to working in small groups on mini–projects. Further into the semester, students must work either by themselves or in pairs on a final project which can either be (i) independent research or (ii) careful reading of an appropriate journal article. A written report must be provided for both of these options, and the topic for research of paper to be read must first be discussed with the instructors. The grade for the course will be based primarily on the result of these projects.

MONDAY	WEDNESDAY	FRIDAY
Jan 7th **1** (TD-Lecture 1) *Basics of probability*	9th **2** (TD-Lecture 2) *Deterministic evolution operators (Liouville)*	11th **3** (TD-Lecture 3) *Stochastic evolution operators (Fokker Planck)*
14th **4** (ST-Lecture 1.a) *Brownian Motion (I)*	16th **5** (ST-Lecture 1.b) *Brownian Motion (II)*	18th **6** Exercise class *Gaussian processes*
21st **7** (ST-Lecture 2) *Stationary processes*	23rd **8** (TD-Lecture 4) *Turbulence*	25th **9** Exercise Class *Mori-Zwanzig*
28th **10** (TD-Lecture 5) *Multi-fractal formalism*	30th **11** (ST-Lecture 3.a) *Stochastic Integration (I)*	Feb 1st **12** Exercise class *Taylor Diffusion*
4th **13** (ST-Lecture 3.b) *Stochastic Integration (II)*	6th **14** (ST-Lecture 3.c) *Stochastic Integration (III)*	8th **15** Exercise class *Ito Calculus*
11th **16** (ST-Lecture 4.a) *Lagrangian transport (I)*	13th **17** (ST-Lecture 4.b) *Lagrangian transport (II)*	15th **18** Exercise Class
18th **19** (ST-Lecture 7) *Kraichnan model as a modelling tool : a selected overview*	20th **20** (ST-Lecture 8) *Beyond diffusive modelling*	22nd **21** Exercise Class

MONDAY	WEDNESDAY	FRIDAY
25th **22** In-class projects	27th **23** Presentations	Mar 1st **24** Presentations