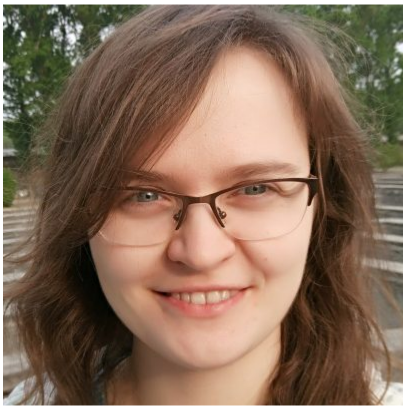
On the impact of diffusion ratio on vanishing viscosity solutions of Riemann problems for chemical flooding models



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We will discuss the vanishing viscosity solutions for a chemical flooding model ($x \in \mathbb{R}$, $t \in \mathbb{R}_+$):

$$s_t + f(s,c)_x = \varepsilon_1 (g_1(s,c) s_x)_x,$$
$$(cs + a(c))_t + (cf(s,c))_x = \varepsilon_1 (c g_1(s,c) s_x)_x + \varepsilon_2 (g_2(s,c) c_x)_x.$$

Here $s = s(x,t)$ — water saturation, $c = c(x,t)$ — concentration of a solvent, $f(s,c)$ — fractional-flow function, $a(c)$ — an adsorption term, $g_1(s,c)$ and $g_2(s,c)$ — diffusion coefficients (separated from zero and infinity), ε_1 and ε_2 are small parameters tending to zero. The model originates from oil industry and describes a process of injection water with solvent into the oil in a porous media. The main message of the talk is the following:

- if $f(s,c)$ is monotone with respect to c, then there is a unique vanishing viscosity solution to the Riemann problem (see e.g. [1], [2]). The typical example is the polymer flooding.
- if $f(s,c)$ is non-monotone with respect to c, then there may be *not unique* vanishing viscosity solution to the Riemann problem. In particular the limit depends on the ratio of diffusion parameters $k = \varepsilon_1/\varepsilon_2$. Some examples are given in [3]. We generalize these examples in [4] and apply the result to surfactant flooding.

References

[1] Johansen T., Winther R. The solution of the riemann problem for a hyperbolic system of conservation laws modeling polymer flooding // SIAM journal on mathematical analysis. — 1988. — Vol. 19, no. 3. — P. 541–566.

[2] Guerra G., Shen W. Vanishing viscosity solutions of riemann problems for models of polymer flooding // To appear in “Partial Differential Equations, Mathematical Physics, and Stochastic Analysis”. A Volume in Honor of Helge Holden’s 60th Birthday. EMS Congress Reports. — 2017.

[3] Shen W. On the uniqueness of vanishing viscosity solutions for riemann problems for polymer flooding // Nonlinear Differential Equations and Applications NoDEA. — 2017. — Vol. 24, no. 4. — P. 37.

[4] Bakharev F., Enin A., Petrova Y., Rastegaev N. On the impact of diffusion ratio on vanishing viscosity solutions of riemann problems for chemical flooding models // Work in progress. — 2020.

Everyone is welcome!